I

A) $z_1 = 5+3i$
$z_2 = 3+2i$

$z_1+z_2 = (8,5)$

$z_1 z_2 = (9,19)$

$\frac{z_1}{z_2} = \left(\frac{21}{13}, \frac{-1}{13}\right)$

$(5,3)(3,-2)$

B) $z_1 = 4+2i$
$z_2 = 5-6i$

$z_1+z_2 = (9,-4)$

$z_1 z_2 = (32,-14)$

$\frac{z_1}{z_2} = \left(\frac{8}{61}, \frac{34}{61}\right)$

error $4+4i$

① $(9,-2)$
② $(44,-4)$
③ $\left(\frac{-4}{61}, \frac{44}{61}\right)$

C) $z_1 = (6,-2)$
$z_2 = (3,4)$

$z_1+z_2 = (9,2)$

$z_1 z_2 = (26,18)$

$\frac{z_1}{z_2} = \left(\frac{2}{5}, \frac{-6}{5}\right)$

D) $z_1 = 2\text{cis}30$ $\frac{2\sqrt{3}}{2}, \frac{2}{2}$
$z_2 = 3\text{cis}60$ $\frac{3}{2}, \frac{3\sqrt{3}}{2}$

$z_1+z_2 = \left(\frac{2\sqrt{3}+3}{2}, \frac{2+3\sqrt{3}}{2}\right)$

$z_1 z_2 = 6\text{cis}90 = (0,6)$

$\frac{z_1}{z_2} = \frac{2}{3}\text{cis}-30 = \frac{2}{3}\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) = \left(\frac{\sqrt{3}}{3}, \frac{-1}{3}\right)$

II

A) $z_1 = -1+i = \sqrt{2}\text{cis}135$
$z_2 = -2+2i = 2\sqrt{2}\text{cis}135$

$z_1 z_2 = 4\text{cis}270$

$\frac{z_1}{z_2} = \frac{1}{2}\text{cis}0$

B) $z_1 = 4-4i = 4\sqrt{2}\text{cis}(-45)$
$z_2 = -2i = 2\text{cis}(270)$

$z_1 z_2 = 8\sqrt{2}\text{cis}225$

$\frac{z_1}{z_2} = 2\sqrt{2}\text{cis}45$

C) $z_1 = 5\sqrt{2}\text{cis}45$
$z_2 = 5(1+i) = 5\sqrt{2}\text{cis}45$

$z_1 z_2 = 50\text{cis}90$

$\frac{z_1}{z_2} = \text{cis}0$

D) $z_1 = 4\text{cis}270$
$z_2 = 4\text{cis}90$

$z_1 z_2 = 16\text{cis}0$

$\frac{z_1}{z_2} = \text{cis}180$

III

$\left(\frac{3\sqrt{3}}{2} + \frac{3i}{2}\right)^{-6} = \left[3\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)\right]^{-6} = (3\text{cis}30)^{-6} = 3^{-6}\text{cis}-180$

$\frac{81}{9}$

729

$\frac{-1}{729} + 0i$

IV $z^4 = i = \text{cis}(90 + 360k)$

$z = \text{cis}\ \frac{90}{4} + 90k$

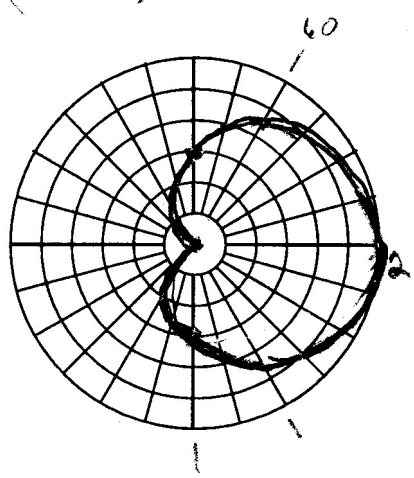
$\text{cis}\ \frac{45}{2} = \text{cis}\ \frac{45}{2}$

$\text{cis}\ \frac{45}{2} + 90 = \text{cis}\ \frac{225}{2}$

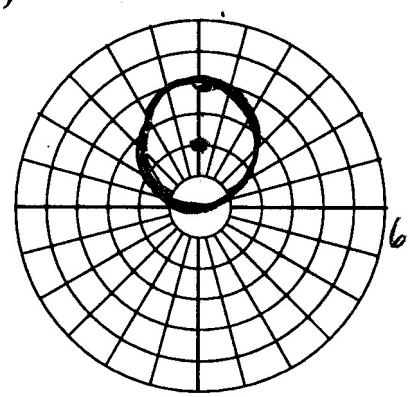
$\text{cis}\ \frac{45}{2} + 180 = \text{cis}\ \frac{405}{2}$

$\text{cis}\ \frac{45}{2} + 270 = \text{cis}\ \frac{585}{2}$

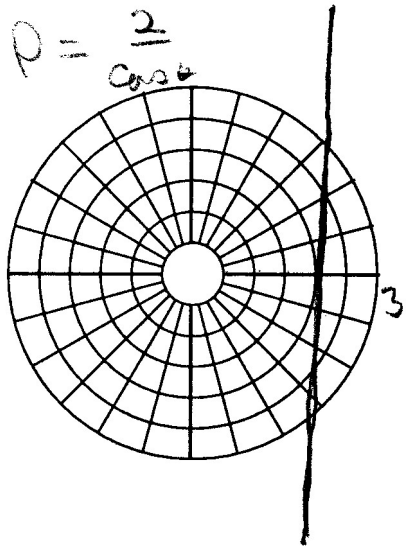
V A) $\rho = 4\cos\theta$



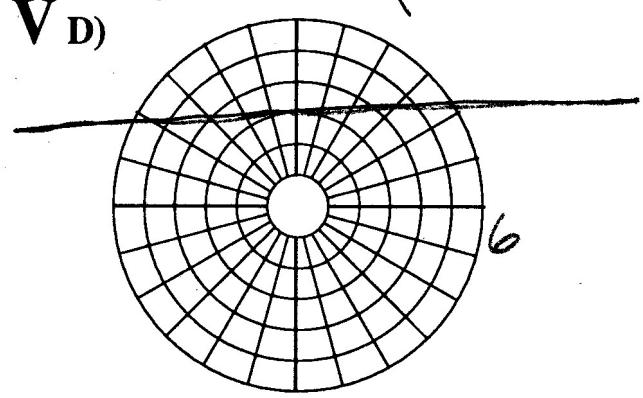
V B) $\rho = 4\sin\theta$



V C) $\rho = \frac{2}{\cos\theta}$



V D) $\sin\theta = \frac{3}{\rho}$



Extra

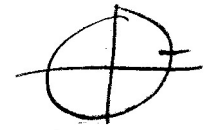
$8\sin\theta\cos^5\theta - 8\sin^5\theta\cos\theta = 1$

$8\sin\theta\cos\theta\,(\cos^4\theta - \sin^4\theta) = 1$

$\underbrace{8\sin\theta\cos\theta}_{4\sin 2\theta}\,\underbrace{(\cos^2\theta - \sin^2\theta)}_{\cos 2\theta}\,\underbrace{(\cos^2\theta + \sin^2\theta)}_{1} = 1$

$2(2\sin 2\theta\cos 2\theta) = 2\sin 4\theta$

$\sin 4\theta = \frac{1}{2}$



$4\theta = 30$

$\theta = \frac{15}{2}$

odd 4/00

	4	5	T
100	1	1	2
90	6	8	14
80	4	3	7
70	3	3	6
60	7	2	9
59	3	4	7

100

50

$\frac{1783}{23}$ $\frac{1690}{21}$

77.5 80.5 78.9

Nothing curved here....

Those who did what they were supposed to get A's and the others failed.

Honors Trigonometry
Copy original problem.
Convince *me* that **you** understand the concept!
You may use a calculator in section V *only*.

Name TRIG Ch 7
Per _____ Date 2009

Chapter 7 Exam

I Given z_1 and z_2 find: $z_1 + z_2$, $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$.

Write your answers in cartesian coordinate form [ie. (a,b)] where a and b are real numbers.

(3 pts ea part; total 36 pts)

A) $z_1 = 5+3i$, $z_2 = 3+2i$

B) $z_1 = 4+\sqrt{-4}$, $z_2 = 5-\sqrt{-36}$

C) $z_1 = (6,-2)$, $z_2 = (3,4)$

D) $z_1 = 2\text{cis}30°$, $z_2 = 3\text{cis}60°$

II Given z_1 and z_2 find: $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$.. (tot 24 pts)

Write your final answer in the form of: $\rho\,\text{cis}\,\theta$; $(0 \le \theta < 360\ ;\ \rho \ge 0)$

A) $z_1 = -1+i$, $z_2 = -2+2i$

B) $z_1 = 4-4i$, $z_2 = -2i$

C) $z_1 = 5\sqrt{2}\ \text{cis}\ 45°$, $z_2 = 5(1+i)$

D) $z_1 = 4\text{cis}270°$, $z_2 = 4\text{cis}90°$

III Express: $\left(\frac{3\sqrt{3}}{2}+\frac{3i}{2}\right)^{-6}$ in the form of $a + bi$ (10 pts)

IV Find the four fourth roots of i. Write your answers in $\rho\,\text{cis}\,\theta$; $(0 \le \theta < 360\ ;\ \rho \ge 0)$ (10 pts)

V Sketch the following equations on the axes provided on the opposite side of this paper. (5 pts ea)
Mark 2 points (which are **not on any axis**) for *each* equation and give the coordinate in both cartesian and polar form.

A) $\rho = 3(1+\cos\theta)$

B) $\rho = 4\sin\theta$

C) $\rho = \frac{2}{\cos\theta}$

D) $\sin\theta = \frac{3}{\rho}$

Extra Credit ••••••••••••••••••••••••••••••••••••• **5 pts** •••

Compute the number of degrees in the smallest positive angle θ such that:

$$8\sin\theta\cos^5\theta - 8\sin^5\theta\cos\theta = 1$$

I

A) $z_1 = 2+3i$
$z_2 = 3-4i$

① $z_1+z_2 = (5,-1)$
② $z_1 z_2 = (18,1)$
③ $\frac{z_1}{z_2} = \left(\frac{-6}{25}, \frac{17}{25}\right)$

B) $z_1 = -5+\sqrt{-16}$
$z_2 = 4-\sqrt{-36}$

① $z_1+z_2 = (-1,-2)$
② $z_1 z_2 = (4,46)$
③ $\frac{z_1}{z_2} = \left(-\frac{11}{13}, -\frac{7}{26}\right)$

C) $z_1 = (4,5)$
$z_2 = (-3,2)$

① $z_1+z_2 = (1,7)$
② $z_1 \cdot z_2 = (-22,-7)$
③ $\frac{z_1}{z_2} = \left(-\frac{2}{13}, \frac{-23}{13}\right)$

D) $z_1 = 2\,\text{cis}\,120$ $(-1,\sqrt{3})$
$z_2 = -3\,\text{cis}\,30$ $\left(\frac{-3\sqrt{3}}{2}, -\frac{3}{2}\right)$

① $z_1+z_2 = \left(\frac{-2-3\sqrt{3}}{2} - \frac{3-2\sqrt{3}}{2}\right)$
Not simple but can -1
② $-6\,\text{cis}\,150 = (3\sqrt{3}, -3)$
③ $-\frac{2}{3}\,\text{cis}\,90 = \left(0, -\frac{2}{3}\right)$

II

A) $z_1 = 2-2i = 2\sqrt{2}\,\text{cis}(-45)$
$z_2 = -1+i = \sqrt{2}\,\text{cis}(135)$

① $z_1 z_2 = 4\,\text{cis}\,90$
② $\frac{z_1}{z_2} = 2\,\text{cis}\,180$

B) $z_1 = (0,-2) = 2\,\text{cis}(-90)$
$z_2 = (4,-4) = 4\sqrt{2}\,\text{cis}(-45)$

① $z_1 z_2 = 8\sqrt{2}\,\text{cis}\,225$
② $\frac{z_1}{z_2} = \frac{\sqrt{2}}{4}\,\text{cis}\,315$

C) $z_1 = 4-4i = 4\sqrt{2}\,\text{cis}(-45)$
$z_2 = 4\sqrt{2}\,\text{cis}\,135$

① $z_1 z_2 = 32\,\text{cis}\,90$
② $\frac{z_1}{z_2} = 1\,\text{cis}\,180$

D) $z_1 = 3\,\text{cis}\,30$
$z_2 = 6\,\text{cis}\,45$

① $18\,\text{cis}\,75$
② $\frac{1}{2}\,\text{cis}\,345$

III

$\left(\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)^{-4}$

$3^{-4}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{-4}$

$3^{-4}\left[\text{cis}(-30)\right]^{-4}$

$3^{-4}\,\text{cis}\,120$

$\frac{1}{81}\,\text{cis}\,120$

$\frac{1}{81}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$-\frac{1}{162} + \frac{\sqrt{3}}{162}i$

Include neg exponents -2

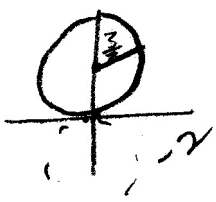
IV

a) $(x+3)^2 + (y-1)^2 = 10$
$x^2+6x+9+y^2-2y+1 = 10$
$x^2+y^2+6x-2y = 0$
$\rho^2 + 6\rho\cos\theta - 2\rho\sin\theta = 0$
$\rho = 2\sin\theta - 6\cos\theta$

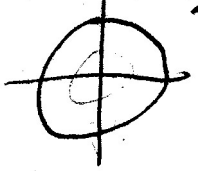
b) $\rho = \frac{6}{2\cos\theta - 3\sin\theta}$
$2\rho\cos\theta - 3\rho\sin\theta = 6$
$2x - 3y = 6$
$3y = 2x - 6$
$y = \frac{2}{3}x - 2$

V

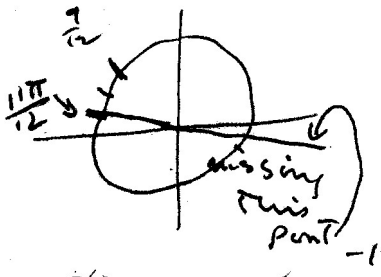
a) $\rho = 3\sin\theta$



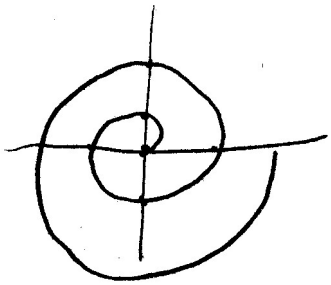
b) $\rho = -2$
circle radius 2



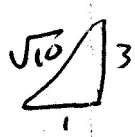
c) $\theta = \frac{11}{12}\pi$



D) $\rho = \frac{\theta}{180}$



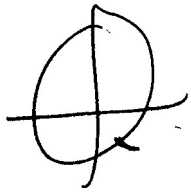
Extra $\sin x = 3\cos x$
$\therefore \tan x = 3$



$\sin x \cos x$
$\frac{3}{\sqrt{10}} \cdot \frac{1}{\sqrt{10}} = \frac{3}{10}$

$2-2i \quad 1-i$

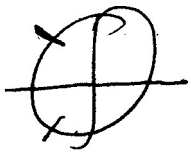
$-1+i$



4) $\sqrt{2}\,\text{cis}\,315$

c)

$4\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$



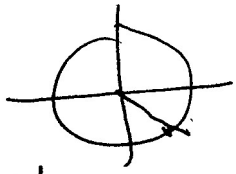
$8-8i \quad 8\sqrt{2}\,\text{cis}\,135$

b)

$0, -2 \quad 4-6$

$(4,-4)$

16
36
52
$4\sqrt{13}$



$\theta = 3\,03^{41}$

d) $\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$\frac{6\sqrt{2}}{2} + \frac{6\sqrt{2}}{2}$

$\frac{3\sqrt{3}+6\sqrt{2}}{2} + \frac{3+6\sqrt{2}}{2}i$

$40^{\circ}\,\underline{9}$

If They Add in II check answers.

$17.9\,\text{cis}\,40$

1/12/2001

	4	5	T
100↑	0	1	1
90↑	3	7	10
80↑	4	6	10
70↑	6	6	12
60↑	4	3	7
59↓	2	4	6

101

27

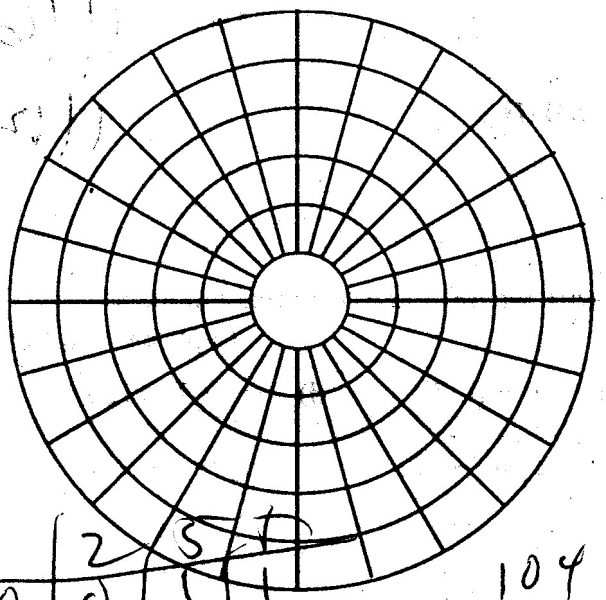
1402 / 19 = 73.8

2125 / 27 = 78.7

96.7

The Names change ...
Nothing else does ...
No changes ... again.

V A)



93

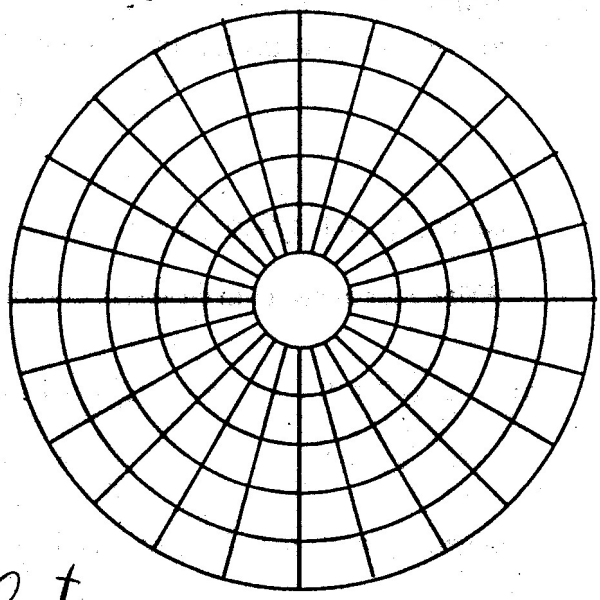
	2	5	7
100	0	1	1
90	9	10	19
80	6	2	8
70	3	7	10
60	2	1	3
59	0	2	2
	1716	1845	3561
	20	22	42

84.8

104

23

V B)

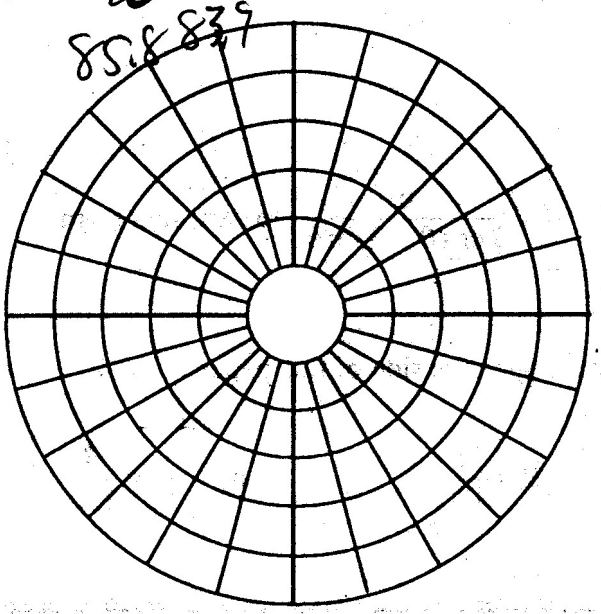


Excellent exam. Excellent unit. make the change I have on extra.

I did remind all to "use proper form" for answers. Generally all observed it.

V C)

85.8 83.9



98

96

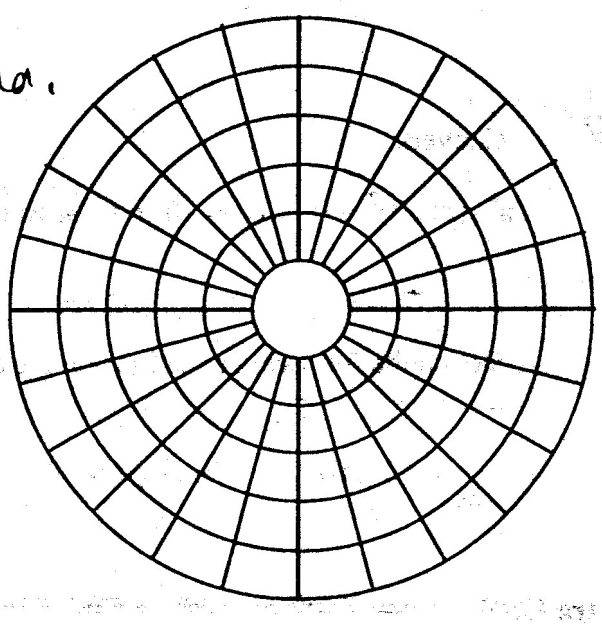
27

100	0
90	12
80	5
70	3
60	1
59	5

2043 / 25

81.7

V D)



as before.....perfect.

Honors Trigonometry
Copy original problem.
Convince *me* that **you** understand the concept!
No Calculators!

Name Chi Pang
Per _____ Date 11/12/2001

Chapter 7 Exam

I Given z_1 and z_2 find: $z_1 + z_2$, $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$. (3 pts ea part; total 36 pts)

Write your answers in cartesian coordinate form [ie. (a,b)] where a and b are real numbers.

a) $z_1 = 2 + 3i$, $z_2 = 3 - 4i$

b) $z_1 = -5 + \sqrt{-16}$, $z_2 = 4 - \sqrt{-36}$

c) $z_1 = (4, 5)$, $z_2 = (-3, 2)$

d) $z_1 = 2 \text{ cis } 120^\circ$, $z_2 = -3 \text{ cis } 30^\circ$

II Given z_1 and z_2 find: $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$ (tot 24 pts)

Write your final answer in the form of: $\rho \text{ cis } \theta$; $(0 \leq \theta < 360 \; ; \; \rho \geq 0)$

a) $z_1 = 2 - 2i$, $z_2 = -1 + i$

b) $z_1 = (0, -2)$, $z_2 = (4, -4)$

c) $z_1 = 4 - 4i$, $z_2 = 4\sqrt{2} \text{ cis } 135^\circ$

d) $z_1 = 3 \text{ cis } 30^\circ$, $z_2 = 6 \text{ cis } 45^\circ$

III Express: $\left(\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)^{-4}$ in the form of $a + bi$ (10 pts)

IV Convert: (5 pts ea)

a) $(x + 3)^2 + (y - 1)^2 = 10$ to polar form

b) $\rho = \frac{6}{2\cos\theta - 3\sin\theta}$ to cartesian form

V Sketch the following equations on the axes provided on the opposite side of this paper. (5 pts ea)

a) $\rho = 3\sin\theta$

b) $\rho = -2$

c) $\theta = \frac{11}{12}\pi$

d) $\rho = \frac{\theta}{180}$

Extra Credit 5 pts

Given: $\sin x = 3\cos x$ and $\sin x \cos x = k$. Solve for k.
(hint: k is a positive number less than 2.)

I TOTAL 30 PTS.

A) $z_1 = -3 + 2i$

$z_2 = 4 - 5i$

$z_1 + z_2 = (1, -3)$

$z_1 z_2 = (-2, 23)$

$\frac{z_1}{z_2} = \left(\frac{-22}{41}, \frac{-7}{41}\right)$

B) $z_1 = 4 + 3\sqrt{2}i$

$z_2 = 5 - 2\sqrt{2}i$

$z_1 + z_2 = (9, \sqrt{2})$

$z_1 z_2 = (32, 7\sqrt{2})$

$\frac{z_1}{z_2} = \left(\frac{8}{33}, \frac{23\sqrt{2}}{33}\right)$

C) $z_1 = (6, -2)$

$z_2 = (-3, 4)$

$z_1 + z_2 = (3, 2)$

$z_1 z_2 = (-10, 30)$

$\frac{z_1}{z_2} = \left(\frac{-16}{25}, \frac{-18}{25}\right)$

d) $z_1 = 2 \operatorname{cis} 210$

$(-\sqrt{3}, -1)$

$z_2 = 3 \operatorname{cis} 300$

$\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

$z_1 + z_2 = \left(\frac{3-2\sqrt{3}}{2}, -\frac{2+3\sqrt{3}}{2}\right)$

$z_1 z_2 = 6 \operatorname{cis} 150$

$(-3\sqrt{3}, 3)$

$\frac{z_1}{z_2} = \frac{2}{3} \operatorname{cis} -90$

$\left(0, -\frac{2}{3}\right)$

II TOTAL 20 PTS

A) $z_1 = -1 + i = \sqrt{2} \operatorname{cis} 135$

$z_2 = -2 + 2i = 2\sqrt{2} \operatorname{cis} 135$

$z_1 z_2 = 4 \operatorname{cis} 270$

$\frac{z_1}{z_2} = \frac{\sqrt{2} \operatorname{cis} 135}{2\sqrt{2} \operatorname{cis} 135} = \frac{1}{2} \operatorname{cis} 0$

B) $z_1 = 4 - 4i = 4\sqrt{2} \operatorname{cis} -45$

$z_2 = -2i = 2 \operatorname{cis} 270$

$z_1 z_2 = 8\sqrt{2} \operatorname{cis} 225$

$\frac{z_1}{z_2} = 2\sqrt{2} \operatorname{cis} 45$

C) $z_1 = 5 \operatorname{cis} 150$

$z_2 = 5(1 - i) = 5\sqrt{2} \operatorname{cis} (-45)$

$z_1 z_2 = 25\sqrt{2} \operatorname{cis} 105$

$\frac{z_1}{z_2} = \frac{\sqrt{2}}{2} \operatorname{cis} 195$

D) $z_1 = 4 \operatorname{cis} 30$

$z_2 = 8 \operatorname{cis} 60$

$z_1 z_2 = 32 \operatorname{cis} 90$

$\frac{z_1}{z_2} = \frac{1}{2} \operatorname{cis} 330$

III

$\left(\frac{3\sqrt{3}}{2} - \frac{3i}{2}\right)^{-5}$

$\left(3\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right)^{-5}$

$(3 \operatorname{cis} -30)^{-5}$

$\frac{1}{3^5} \operatorname{cis} 150$

$\frac{1}{243}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$\frac{-\sqrt{3}}{486} + \frac{1}{486}i$

IV

$z^4 = i = \operatorname{cis}(90 + 360k)$

$z = \operatorname{cis}\left(\frac{45}{2} + 90k\right)$

$z = \operatorname{cis} \frac{45}{2}$

$z = \operatorname{cis} \frac{225}{2}$

$z = \operatorname{cis} \frac{405}{2}$

$z = \operatorname{cis} \frac{585}{2}$

V on white paper

Extra, $rt = d$

Let D = Dist from Sacto to Fresno

$\frac{D}{40}$ = Time to Fresno

$\frac{D}{60}$ = Time from Fresno

$\frac{D}{40} + \frac{D}{60}$ = Total Time for Trip

2.20 3.20

2D Total Distance

$\frac{3D + 2D}{6.20} = \frac{5D}{6.20} = \frac{D}{24}$ = Total Time

$r = \frac{D}{t}$ ∴ $\frac{2D}{\frac{D}{24}} = 48$ ∴

Aver speed 48 mph

Honors Trigonometry
Copy original problem.
Convince *me* that **you** understand the concept!
You may use a calculator in section V *only*.

Name TRIG Ch 7

Per _____ Date 1/11/02

Chapter 7 Exam

I Given z_1 and z_2 find: $z_1 + z_2$, $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$.

Write your answers in cartesian coordinate form [ie. (a,b)] where a and b are real numbers.

(3 pts ea part; total 36 pts)

A) $z_1 = -3 + 2i$, $z_2 = 4 - 5i$

B) $z_1 = 4 + \sqrt{-18}$, $z_2 = 5 - \sqrt{-8}$

C) $z_1 = (6, -2)$, $z_2 = (-3, 4)$

D) $z_1 = 2\,\text{cis}\,210°$, $z_2 = 3\,\text{cis}\,300°$

II Given z_1 and z_2 find: $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$. (tot 24 pts)

Write your final answer in the form of: $\rho\,\text{cis}\,\theta$; $(0 \le \theta < 360\ ;\ \rho \ge 0)$

A) $z_1 = -1 + i$, $z_2 = -2 + 2i$

B) $z_1 = 4 - 4i$, $z_2 = -2i$

C) $z_1 = 5\text{ cis }150°$, $z_2 = 5(1 - i)$

D) $z_1 = 4\,\text{cis}\,30°$, $z_2 = 8\,\text{cis}\,60°$

III Express: $\left(\frac{3\sqrt{3}}{2} - \frac{3i}{2}\right)^{-5}$ in the form of $a + bi$ (10 pts)

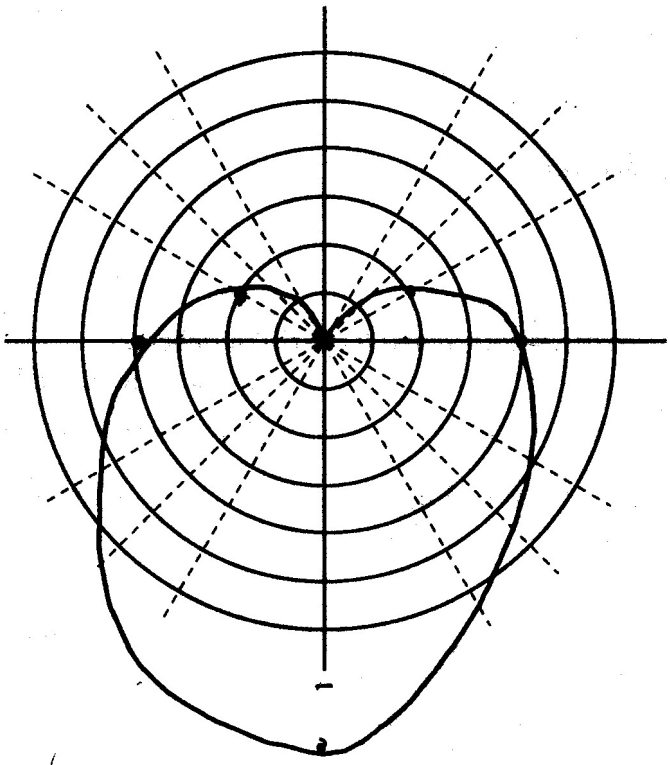
IV Find the four fourth roots of i. Write your answers in $\rho\,\text{cis}\,\theta$; $(0 \le \theta < 360\ ;\ \rho \ge 0)$ (10 pts)

V Sketch the following equations on the axes provided on the opposite side of this paper. (5 pts ea)

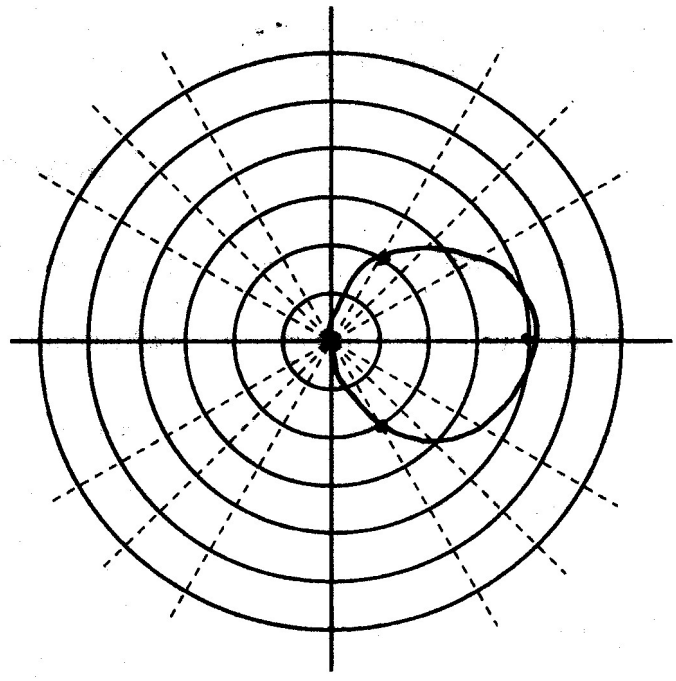
A) $\rho = 4 - 4\sin\theta$ B) $\rho = 4\cos\theta$ C) $\rho = \frac{-2}{\cos\theta}$ D) $\cos\theta = \frac{3}{\rho}$

Extra Credit ••••••

V A)

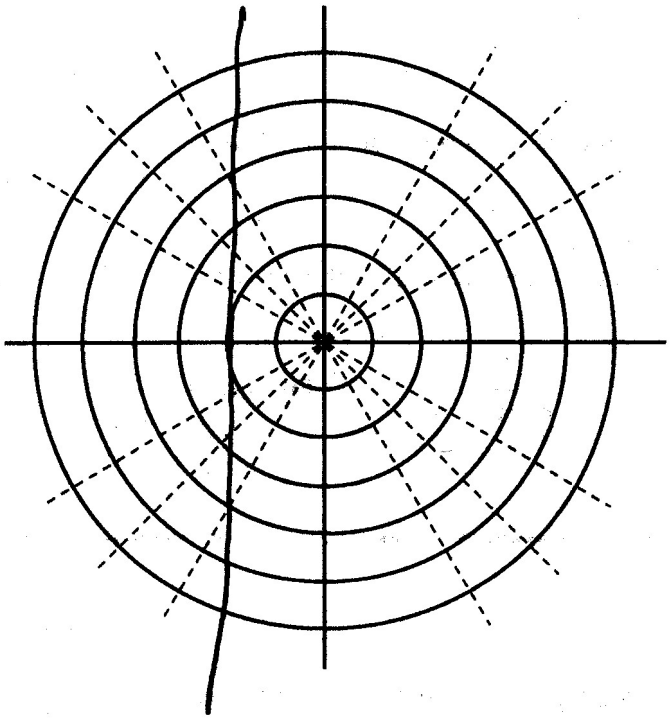


V B)



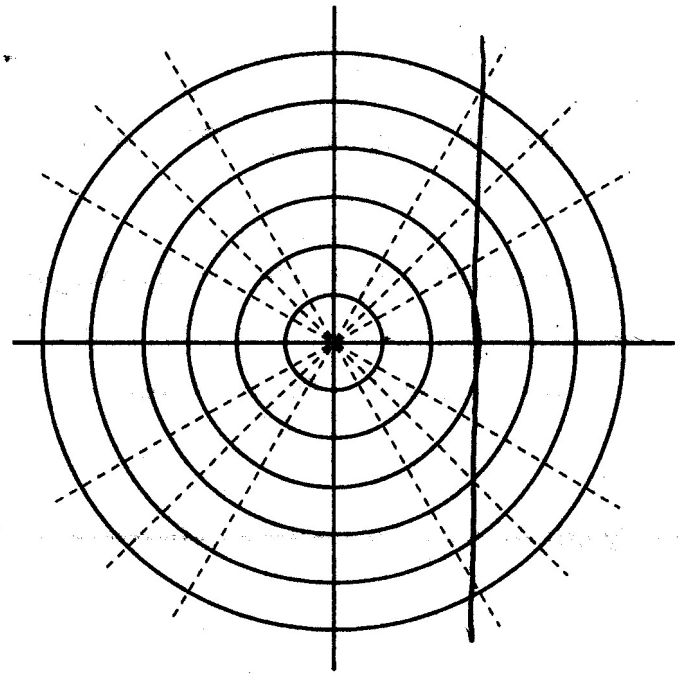
5 pts ea.

V C)

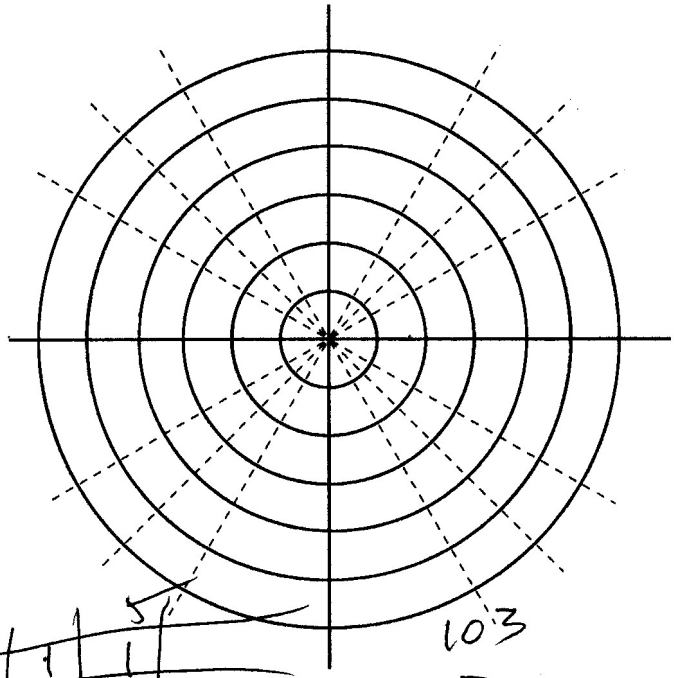


V D)

x = 3



V A)

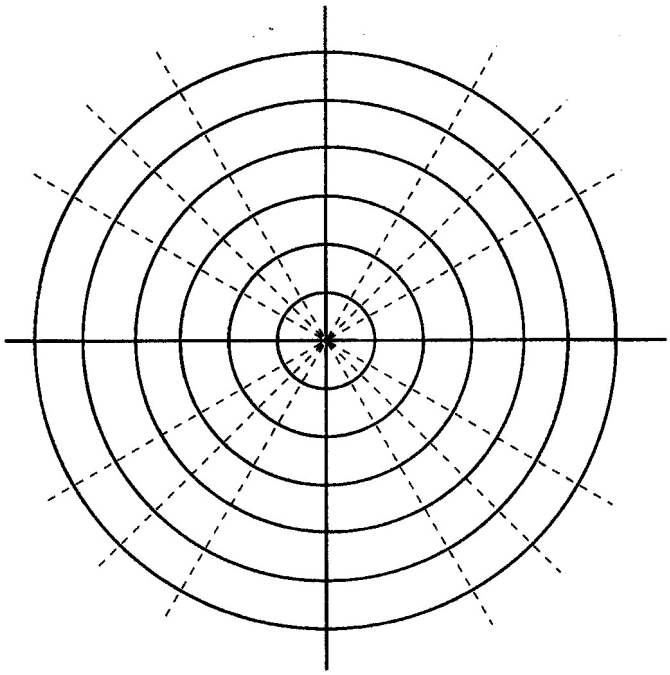


		5
100	1	1
90	5	5
80	6	5
70	5	6
60	3	5
59	3	6

103

43

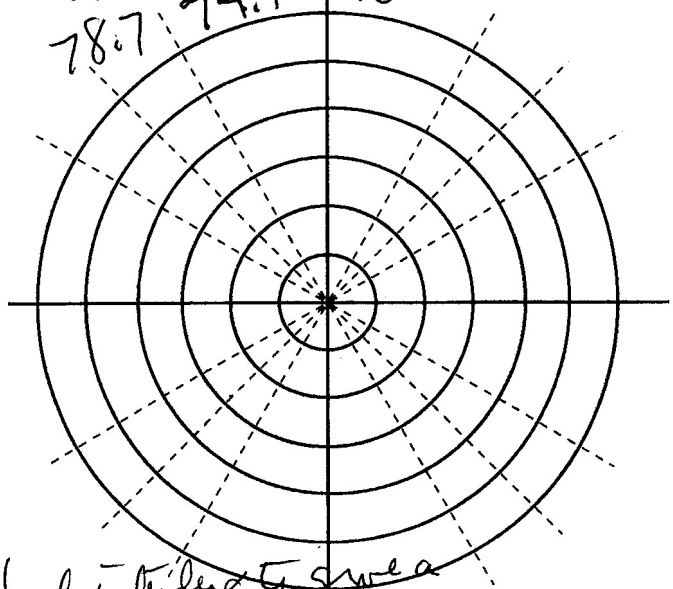
V B)



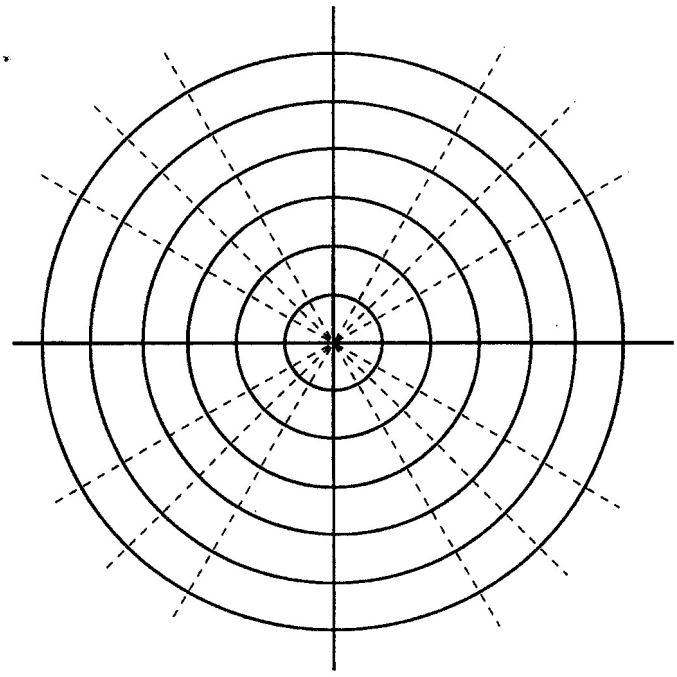
V C)

$\frac{1809}{23}$ $\frac{2098}{28}$ $\frac{3907}{51}$

78.7 74.9 76.6



V D)

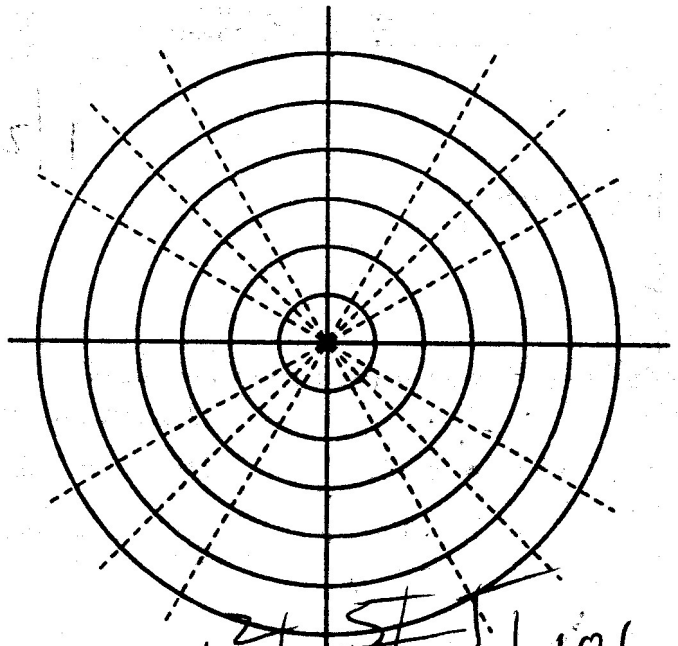


I had intended to give a more difficult version I guess its a good thing I didn't,

MOST wasted time during vacation.

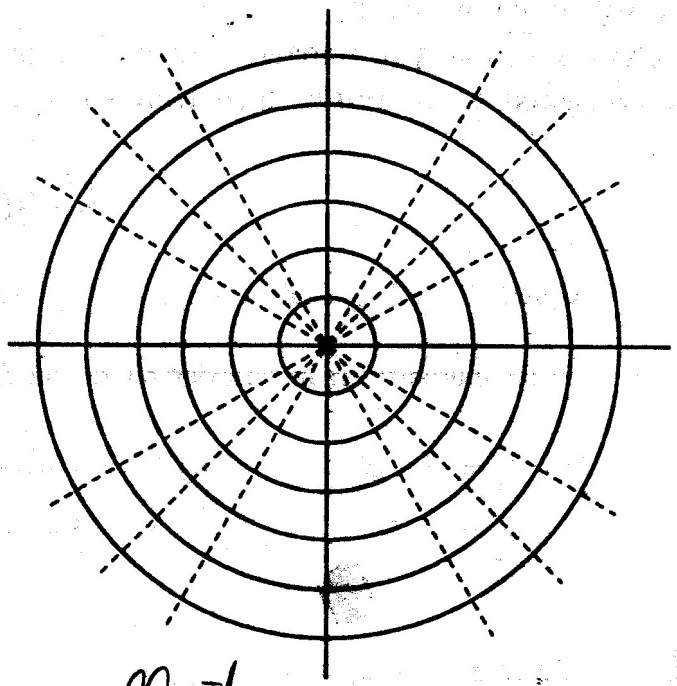
carelessness and laziness account for most of the low scores. This exam was not [illegible] different than [illegible]

V A)



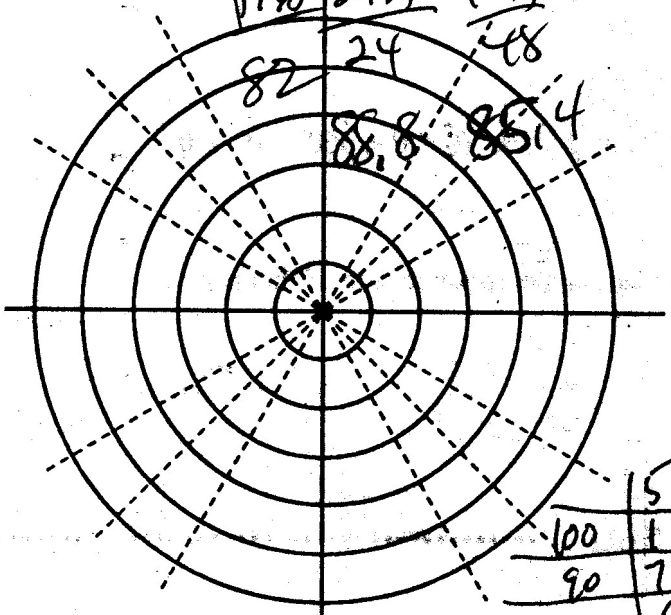
	2	5	7	
100	1	3	4	101
90	11	12	23	
80	1	6	7	
70	7	1	8	
60	2	0	2	
59	2	2	4	41
	1988	2131	4099	
	82	24	48	
		88.8	85.4	

V B)



Excellent exam. change ID to be $z_1 = 2 \text{ cis } 210$, $z_2 = 3 \text{ cis } 330$ much cleaner.

V C)

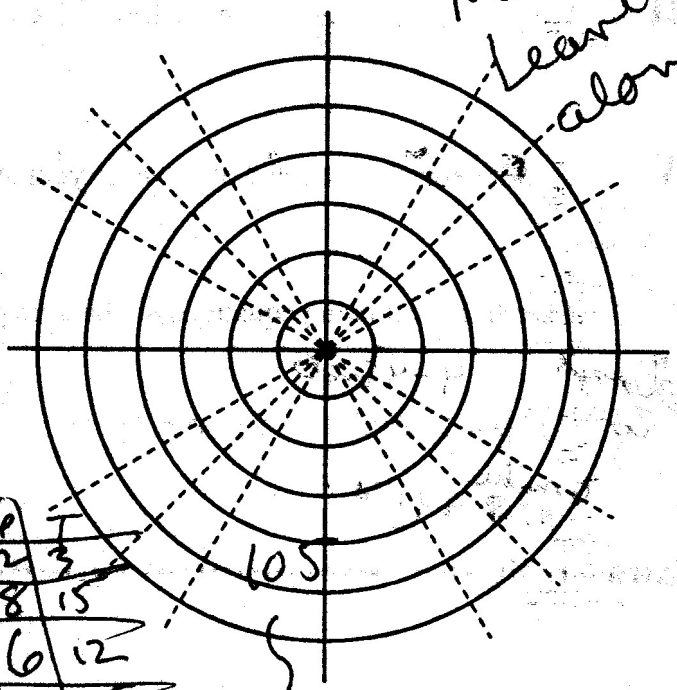


	5	6	7
100	1	2	3
90	7	8	15
80	6	6	12
70	3	2	5
60	2	0	2
59	3	1	4
	82.1	88.5	85

$\frac{1889}{23}$ $\frac{1681}{19}$

V D)

Now — Leave it alone



105

45

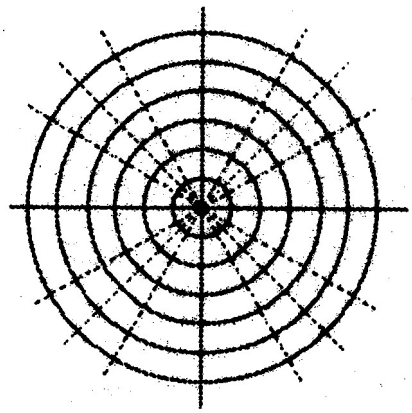
100	1
90	13
80	10
70	3
60	0
59	1

100

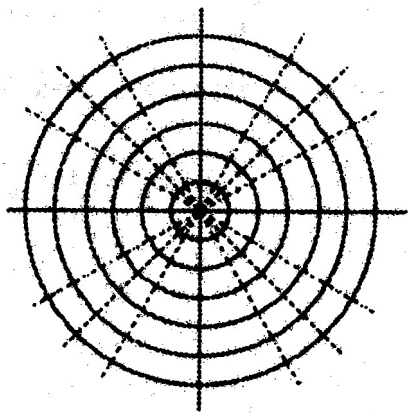
56

2 errors (TYPOS)
Didn't cause any real troubles.
STILL, a great closing exam

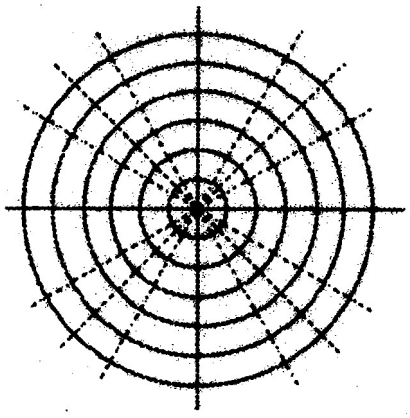
V A) 87.6



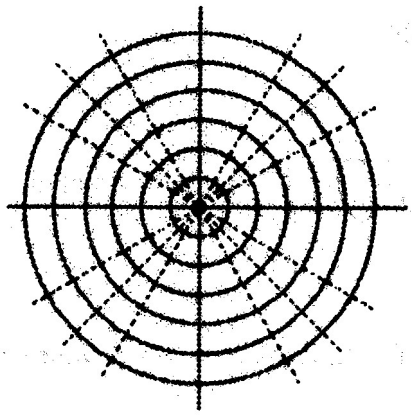
V B)



V C)



V D)



Charlton does Polar

I A On your first polar graph locate each point. Rewrite each coordinate into its cartesian form.

1) $(1,\pi/6)$ 2) $(2,\pi/3)$ 3) $(2,-\pi/3)$ 4) $(-2,-\pi/3)$ 5) $(-2,\pi/3)$
6) $(3,\pi/4)$ 7) $(5,5\pi/6)$ 8) $(-5,11\pi/6)$ 9) $(4,3\pi/4)$ 10) $(3,4\pi/3)$

B On your second polar graph locate each point. Rewrite each coordinate into its cartesian form.

1) $(4,120^\circ)$ 2) $(5,75^\circ)$ 3) $(-3,270^\circ)$ 4) $(2,150^\circ)$ 5) $(3,315^\circ)$
6) $(-4,135^\circ)$ 7) $(3,60^\circ)$ 8) $(4,45^\circ)$ 9) $(5,300^\circ)$ 10) $(4,180^\circ)$

C Rewrite each coordinate into its polar form. Plot on your third polar graph

1) $(3,4)$ 2) $(-3,4)$ 3) $(3,-4)$ 4) $(4,3)$ 5) $(-4,-3)$
6) $(3/2,-3\sqrt{3}/2)$ 7) $(-2,2\sqrt{3})$ 8) $(0,-2)$ 9) $(3\sqrt{2}/2,-3\sqrt{2}/2)$ 10) $(\frac{1}{2},\frac{-\sqrt{3}}{2})$

D Rewrite each coordinate into its polar form. Plot on your fourth polar graph.

1) $(\frac{-3}{2},\frac{-3\sqrt{3}}{2})$ 2) $(2\sqrt{2},-2\sqrt{2})$ 3) $(3,5)$ 4) $(-2,4)$ 5) $(-4,-3)$
6) $(\frac{1}{2},4)$ 7) $(5,-2)$ 8) $(0,-2)$ 9) $(2,3)$ 10) $(\frac{1}{2},\frac{1}{2})$

II For each make a table. Sketch the graph. Convert equation to cartesian form. (2 equations per polar graph.)

a) $r=3\sin\theta$ b) $r=2\csc\theta$ c) $r=\dfrac{3}{4\cos\theta+5\sin\theta}$
d) $r=4\cos\theta+6\sin\theta$ e) $r=2\sin 2\theta$ f) $r=1+\sin\theta$

III Transform each of the following into polar form (r = ...). Sketch on polar graph (2 per axis)

a) $y=x^2$ b) $x+2y=3$ c) $x^2+y^2=3$ d) $xy=2$
e) $x^2+y^2=4x$ f) $x=-2$ g) $y=3$ h) $y=2x+1$

IV a) State at least three pairs of polar coordinates (r,θ) for $(3,3\pi/4)$ where

1) $r>0$ 2) $r<0$

b) Sketch the curves $r=1+\cos\theta$ and $r=1-\sin\theta$ on the same axis. What are the ***cartesian*** coordinates where the two curves intersect? Explain your methods. Explain why there are not three points.

V Make a table of values from zero to 2π for each. Plot the points and sketch on a polar axis:

a) $r=1+2\cos\theta$ b) $r=1+\frac{1}{2}\cos\theta$ c) $r=1-\cos\theta$ d) $r=1-2\cos\theta$

Remember: $z_1 = a_1 + b_1 i$ $z_2 = a_2 + b_2 i$ If $z_1 = z_2$ then $a_1 = a_2$ and $b_1 = b_2$	EXAM Chapter 5 • • • • • • • • • • • • • • R/C Pg 245 - 247 # 1 • • • • • • • • • • • • • • Pg 248 1 - 8 # 2 • • • • • • • • • • • • • •	Short Lecture # 3 Pg 248-249 9 - 35 odd # 4 Pg 245 7 - 19 12/18	Lecture & Demos # 5 "Charlton Does Polar" I 4 graphs II 3 graphs (1 per page) 12/19	Minimum Day **Pick up** **# 6, 7, 8, 9, 10** **"12 Trig Daze of Christmas"** 12/20

Have a *Very* Cool Yule !!!

# 11 "Charlton Does Polar" III 4 graphs IV 1 graph V 4 graphs 1 / 6	# 12 Pg 229 1 — 12 Write the cartesian equation for each polar equation. Locate and label 2 coordinates in cartesian notation on your polar graph which are **NOT** axis intercepts. Need 6 graphs; 2 pblms per graph No tables required 1 / 7	# 13 Pg 230 14 – 16, 19, 21, 23, 24 Write the cartesian equation for each polar equation Needs 7 graphs 1 / 8	# 14 Pg 257-258 1 – 27 1 /9	# 15 Pg 260 1 - 14 Pg 264 2 - 8 even Pg 266 1 - 9 1 /10
# 16, 17, 18, 19,20 "*Imagine That!*" 1 / 13	Continue working on "*Imagine That!*" Golden State Trig Exam 1 / 14	1 / 15	**Put last year's exam on the boards. Come to class prepared!** 1 /16	**Exam Chapter 7** 1 /17
1/ 20	Exams Returned Finish Prep for Final Exam 1/ 21	Final Exam Per 1, 2 1/22	Final Exam Per 3, 4 1/23	Final Exam Per 5, 6 1/24

Honors Trigonometry Chapter 7

2002–2003

I A) $z_1 = 2+3i$

$z_2 = -4+3i$

$z_1+z_2 = (-2,6)$

$z_1 z_2 = (-17,-6)$

$\frac{z_1}{z_2} = \frac{(2,3)}{(-4,3)}\frac{(-4,-3)}{(-4,-3)}$

$= \left(\frac{1}{25}, \frac{-18}{25}\right)$

wrong format I-6

B) $z_1 = 5-\sqrt{-16}$ $\quad 5-4i$

$z_2 = 4+\sqrt{-36}$

$4+6i$

$z_1+z_2 = (9,2)$

$z_1 z_2 = (44,14)$

$\frac{z_1}{z_2} = \frac{(5,-4)}{(4,6)}\frac{(4-6)}{(4,-6)}$

$= \frac{-4}{52} - \frac{46}{52}i$

$= \left(-\frac{1}{13}, \frac{-23}{26}\right)$

C) $z_1 = (4,-5)$

$z_2 = (3,-2)$

$z_1+z_2 = (7,-7)$

$z_1 z_2 = (2,-23)$

$\frac{z_1}{z_2} = \frac{(4,-5)}{(3,-2)}\frac{(3,2)}{(3,2)}$

$= \left(\frac{22}{13}, -\frac{7}{13}\right)$

D) $z_1 = 3\,\text{cis}\,150^\circ$

$z_2 = -2\,\text{cis}\,-60^\circ$

$z_1 \quad -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$z_2 \quad -1+\sqrt{3}i$

$z_1+z_2 = \left(\frac{-3\sqrt{3}-2}{2}, \frac{3+2\sqrt{3}}{2}\right)$

$z_1 z_2 = -6\,\text{cis}\,90$

$(0,-6)$

$\frac{z_1}{z_2} = -\frac{3}{2}\,\text{cis}\,210$

$-\frac{3}{2}\ \frac{-\sqrt{3}}{2}\quad -\frac{3}{2}\ \frac{-1}{2}$

$\left(\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)$

120, -45, 75, 165

II

A) $z_1 = 3-3i = 3\sqrt{2}\,\text{cis}\,-45$

$z_2 = -2+2i \quad 2\sqrt{2}\,\text{cis}\,135$

$z_1 z_2 = 12\,\text{cis}\,90$

$\frac{z_1}{z_2} = \frac{3}{2}\,\text{cis}\,180$

B) $z_1 = (-2,2) = 2\sqrt{2}\,\text{cis}\,135$

$z_2 = (\sqrt{3},-1) = 2\,\text{cis}(-30)$

$z_1 z_2 = 4\sqrt{2}\,\text{cis}\,105$

$\frac{z_1}{z_2} = \sqrt{2}\,\text{cis}\,165$

C) $z_1 = 3\,\text{cis}\,-240$

$z_2 = \frac{5}{\sqrt{2}}\ \frac{-5}{\sqrt{2}} =$

$5\,\text{cis}(-45)$

$z_1 z_2 = 15\,\text{cis}\,-285$

$15\,\text{cis}\,75$

D) $z_1 = 3\sqrt{2}\,\text{cis}\,210$

$z_2 = 2\sqrt{3}\,\text{cis}\,120$

$z_1 z_2 = 6\sqrt{6}\,\text{cis}\,330$

$\frac{z_1}{z_2} = \frac{3\sqrt{2}}{2\sqrt{3}}\,\text{cis}\,90$

$\frac{\cancel{3}\sqrt{6}}{\cancel{6}\,2}$

$\frac{\sqrt{6}}{2}\,\text{cis}\,90$

III $(2,-2\sqrt{3})^3 \quad \left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)^2$

$\left[4\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\right]^3 \quad \left[3\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\right]^2$

$4^3\,\text{cis}\,3(-60) \quad 3^2\,\text{cis}\,2\cdot 120$

$64\cdot 9\,\text{cis}\,(-180+240)$

$576\,\text{cis}\,60$

$576\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$288+288i\sqrt{3}$

$\frac{z_1}{z_2} =$ 120, 45, 75

$\frac{3}{5}\,\text{cis}\,165$

Extra.

$ax^2+bx+c=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2A}$

$z_1 = \frac{-b+\sqrt{b^2-4ac}}{2A}$

$z_2 = \frac{-b-\sqrt{b^2-4ac}}{2A}$

$z_1+z_2 = \frac{-2b}{2A} = -\frac{b}{A}$

IV on Back of Exam

$z_1 \cdot z_2 = \frac{b^2-(b^2-4ac)}{4A^2}$

$= \frac{b^2-b^2+4ac}{4A^2}$

$= \frac{c}{A}$

Honors Trigonometry
Copy original problem.
Convince *me* that **you** understand the concept!
No calculators.

Name Trig
Per _____ Date 2003

Chapter 7 Exam

I Given z_1 and z_2 find: $z_1 + z_2$, $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$.

Write your answers in cartesian coordinate form [ie. (a,b)] where a and b are real numbers.

(3 pts ea part; total 36 pts)

A) $z_1 = 2+3i$, $z_2 = -4+3i$

B) $z_1 = 5-\sqrt{-16}$, $z_2 = 4+\sqrt{-36}$

C) $z_1 = (4,-5)$, $z_2 = (3,-2)$

D) $z_1 = 3\,\text{cis}\,150°$, $z_2 = -2\,\text{cis}(-60°)$

II Given z_1 and z_2 find: $z_1 \cdot z_2$, and $\frac{z_1}{z_2}$. (tot 24 pts)

Write your final answer in the form of: $\rho\,\text{cis}\,\theta$; $(0 \le \theta < 360\ ;\ \rho \ge 0)$

A) $z_1 = 3-3i$, $z_2 = -2+2i$

B) $z_1 = (-2,2)$, $z_2 = (\sqrt{3},-1)$

C) $z_1 = 3\,\text{cis}\,-240°$, $z_2 = \left(\frac{5}{\sqrt{2}}, \frac{-5}{\sqrt{2}}\right)$

D) $z_1 = 3\sqrt{2}\,\text{cis}\,210°$, $z_2 = 2\sqrt{3}\,\text{cis}\,120°$

III Express the product of these two complex numbers: $(2,-2\sqrt{3})^3\left(-\frac{3}{2},\frac{3\sqrt{3}}{2}\right)^2$ in the form of $a + bi$ (10 pts)

IV Fill in the supplied tables (on back of this paper). Plot *all* the points. Sketch the curve on the supplied axis. Rewrite the curve in cartesian format. (hint: $\sqrt{2} \approx 1.4$ and $\sqrt{3} \approx 1.6$) (15 pts ea)

A) $\rho = 6\cos\theta$

B) $\rho = 4(1-\sin\theta)$

Extra Credit ••••••

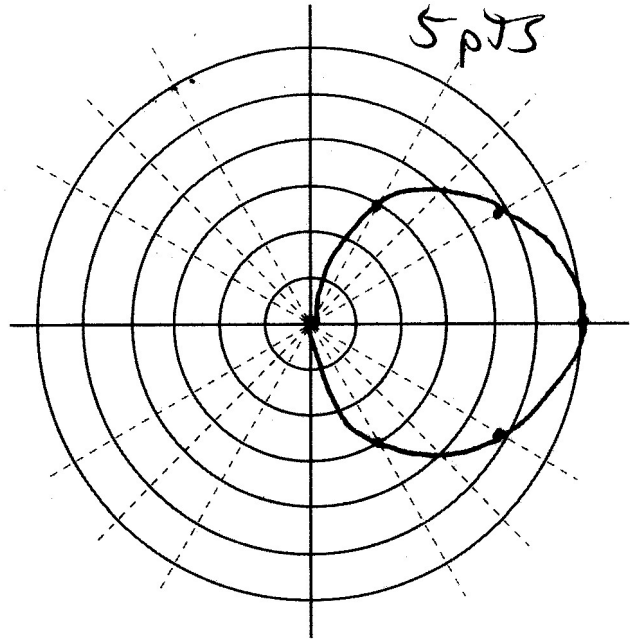
IV A) $\rho = 6\cos\theta$

5 pts

5 pts

$x^2 + y^2 = 6x$

ρ	θ
6	0
4.8	30
3	60
0	90
−3	120
−4.8	150
−6	180
−4.8	210
−3	240
0	270
3	300
4.8	330

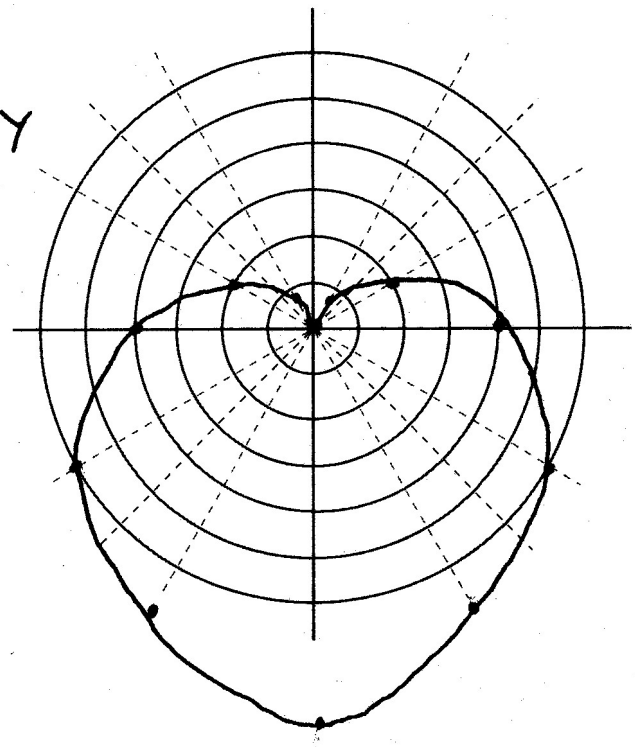


258 28 381

IV B) $\rho = 4(1 - \sin\theta)$

$x^2 + y^2 = 4\sqrt{x^2+y^2} - 4y$

ρ	θ
4	0
2	30
.8	60
0	90
.8	120
2	150
4	180
6	210
7.5 ~~6.5~~	240
8	270
7.5 ~~6.5~~	300
6	330



	2	3	T
100	1	0	1
90	3	2	5
80	5	3	8
70	4	6	10
60	4	2	6
59	7	13	20

100

13

$\frac{1596}{24}$ $\frac{1582}{26}$

66.5 60.8 63.6

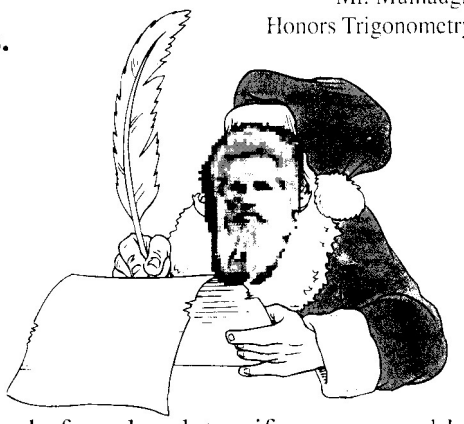
Incredible!!!
These people thought they could use calculators.
OOPS - should have learned the concepts.
Excellent extra
No changes

Mr. Mumaugh
Honors Trigonometry

Santa Heston ...

...has lots of treasures for all the good little boys and girls.

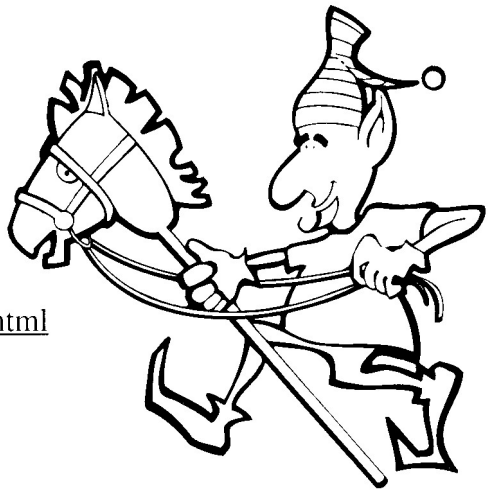
You were good this past year so you will find many presents under the internet tree for you!



This assignment is worth 5 homework assignments
(and up to an additional 5 homework assignments as extra credit.)

Your responses will be graded on accuracy, spelling, and "presentation". (The "presentation" is the paper you turn in. It should be typed or very legibly written. It may contain graphics and/or hand drawings as appropriate and must be turned in on the due date (or postmarked on a day before due date - if you are unable to submit your paper in person.)

A: From the URL list in the 12 Trig Daze of Christmas, select 3 of the 12. Do the chore described for the specific URL.

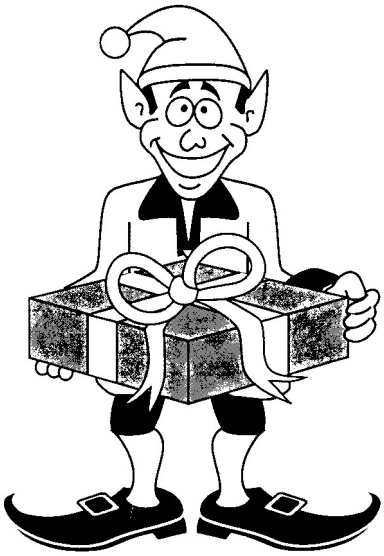


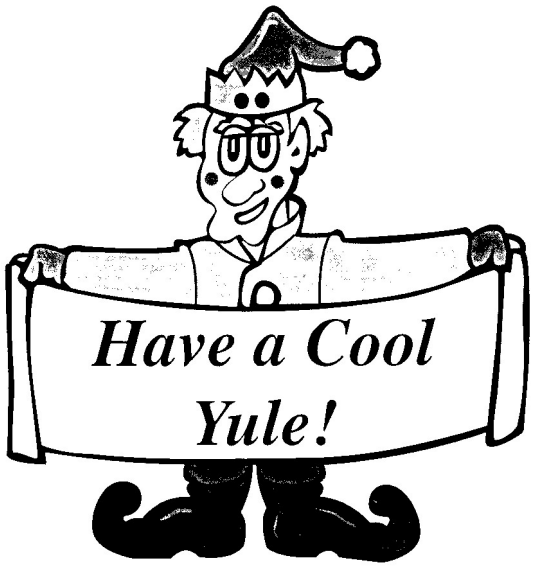
B: 1) Go to : http://www.refdesk.com/homework.html
Click on "Grades 9-12
Click on "Math Homework Help"
Click on "Practice Problems"
Scroll down to "more Trig Identities".
Select 2 identities you found and prove them.

2) Go to: http://aleph0.clarku.edu/~djoyce/java/trig/oblique.html
Scroll down until you find problems #553 - #612.
Supply complete solutions for 2 of the problems.

C: Go to: www.dogpile.com
type in the following "search criteria": +triangle +trig +proof
Select 2 sites which are not in the 12 Trig Daze of Christmas. State the URL and describe what you found that applies to our course.

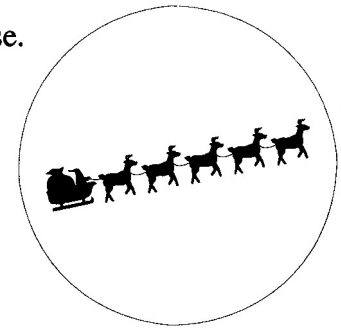
D: You may select up to 5 more URLs from 12 Trig Daze of Christmas as in section A above. Each one completed is extra credit.





12 Trig Daze of Christmas

1. http://shell.rmi.net/~tph/fractalkit/fractal.html
   Select one of the three main links and describe what you found.
2. http://forum.swarthmore.edu/johnandbetty/
   Go to the site and read the story of John and Betty.
   Give a summary of the story.
3. http://www.yourdictionary.com/
   Find out how to spell: "trigonometry" in 10 different languages. Identify the language and write the spelling.
4. http://www.inwit.com/inwit/funstuff.html
   Scroll down until you find: History and Oddities of the Number Pi.
   Describe one of the oddities presented which you found interesting.
5. http://archives.math.utk.edu/wg/wgindex.html
   Type "trigonometry" (without the quotes) in the area: "String to search for". Click on "search".
   Scroll down and click on: Mathematics Archives - Topics in Mathematics - Trigonometry, Jan 6 2000"
   Locate and click on: "Frequently Asked Questions About Trigonometry HS Int Java"
   Read down and click on the links beginning with "Stick and Shadows (part 1)" and ending with "A Few Final Words".
   Describe the material you found.
6. http://forum.swarthmore.edu/pow/
   Locate the section: "Trig & Calculus".
   Select one trigonometry problem which you like. State the problem and give your solution to the problem or critique one of the supplied solutions.
7. http://www-history.mcs.st-and.ac.uk/history/HistoryTopics.html
   Select one of the 38 topics.
   Give a summary of the topic you selected.
8. http://thewizardofodds.com/math
   Select one problem involving trigonometry.
   Describe the problem and its complete solution.
9. http://www.sosmath.com/trig/trig.html
   Locate "Trigonometric Equations"
   Select a problem.
   Describe the problem and its complete solution.
10. http://www.ies.co.jp/math/java/index.html
    Select one of the trig applets and describe what it demonstrates.
11. http://jwilson.coe.uga.edu/emt725/EMT725.html
    Scroll down until you find "trigonometry problems".
    Select one of the 5 links listed. Describe what you find.
12. http://www.ies.co.jp/math/indexeng.html
    Click on "Manipula Math with Java (English)"
    Scroll down to find an applet which relates to our course.
    Describe what you find.



Charlie's Turkey

Charleton (some know him as Santa) Heston delivered turkeys to three students living in Vacaville. He delivered only one turkey on each trip and he always got his turkeys at the same place. "Just for fun", he took three different routes. Because he didn't want too much fun, he always drove at the same speed. (He had a cruise control.) See the map and six facts for assistance.

The question is: How fast did Charlie drive?

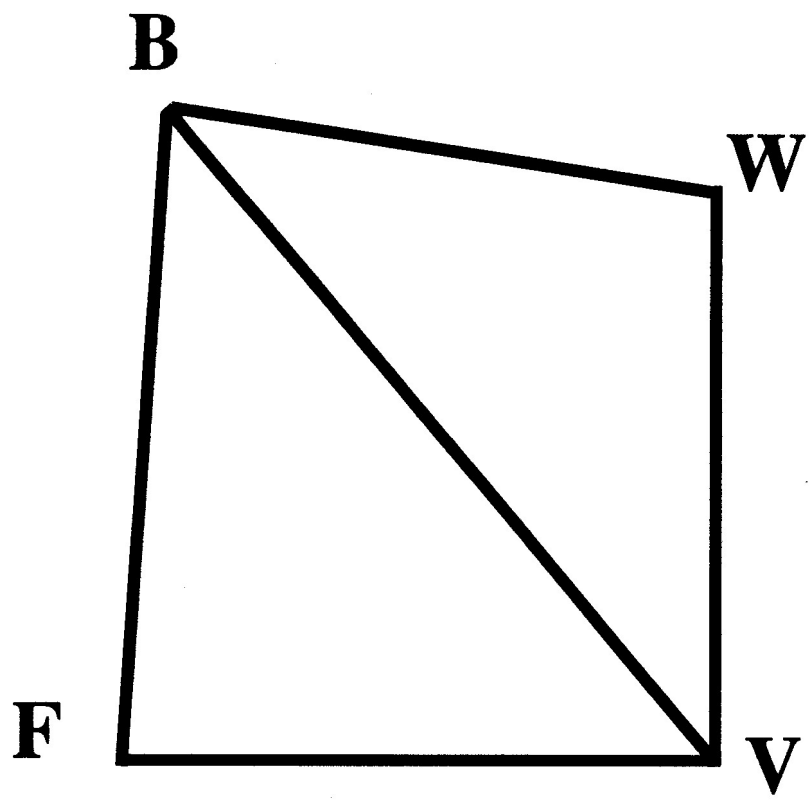
The answer to the question is due the first day we return from Christmas vacation. If you will be delayed, I will accept a solution postmarked on or before that first day we return.

Hint: This *is* enough information to answer the question. Use that new calculator that Santa brought you!

Value: 3 bonus assignments!

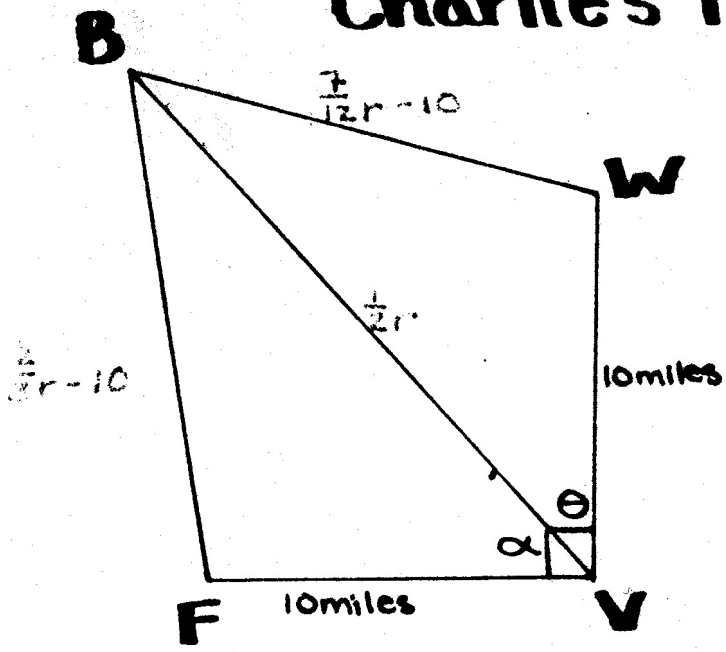
"FACTS"

1. He made it from B directly to V in 30 minutes.
2. He made it from B through W to V in 35 minutes.
3. He made it from B through F to V in 40 minutes.
4. The road V F is east - west and 10 miles long.
5. The road V W is north - south and 10 miles long.
6. All roads are perfectly straight and no speed or time is lost on any turns.



Theresa Allen
per. 5
12-28-8

1. # Charlie's Turkeys



BWV takes 35 minutes or $\frac{7}{12}$ of an hour

B-V takes 30 minutes or $\frac{1}{2}$ of an hour

B-F-V takes 40 minutes or $\frac{2}{3}$ of an hour

Distance = Rate • Time

$d = rt$

$d(BWV) = \frac{7}{12}r$

$d(BV) = \frac{1}{2}r$

$d(BFV) = \frac{2}{3}r$

$d(BW) = d(BWV) - d(WV)$

$d(BF) = d(BFV) - d(FV)$

$d(BW) = \frac{7}{12}r - 10$

$d(BF) = \frac{2}{3}r - 10$

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ θ is also $\left(\frac{\pi}{2} - \alpha\right)$

$\sin^2\left(\frac{\pi}{2} - \alpha\right) + \cos^2\theta = 1$

$\sin\left(\frac{\pi}{2} - \alpha\right)^2 + \cos^2\theta = 1$

$\left(\underbrace{\sin\frac{\pi}{2}}_{1}\cos\alpha - \sin\alpha\underbrace{\cos\frac{\pi}{2}}_{0}\right)^2 + \cos^2\theta = 1$

$\boxed{\cos^2\alpha + \cos^2\theta = 1}$

$\cos^2\alpha$ and $\cos^2\theta$ are found out on the next page

Theresa Allen
per. 5
12-28-87

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\left(\frac{2}{3}r - 10\right)^2 = \left(\frac{1}{2}r\right)^2 + 10^2 - 2\left(\frac{1}{2}r\right)(10)\cos\alpha$$

$$\frac{4}{9}r^2 - \frac{40}{3}r + \cancel{100} = \frac{1}{4}r^2 + \cancel{100} - 10r\cos\alpha$$

$$\frac{4}{9}r^2 - \frac{1}{4}r^2 - \frac{40}{3}r = -10r\cos\alpha$$

$$\frac{16}{36}r^2 - \frac{9}{36}r^2 - \frac{40}{3}r = -10r\cos\alpha$$

$$\frac{7}{36}r^2 - \frac{40}{3}r = -10r\cos\alpha$$

$$-\frac{7r^2}{360} + \frac{40r}{30} = r\cos\alpha$$

$$\frac{-7r}{360} + \frac{40}{30} = \cos\alpha$$

$$\frac{-7r}{360} + \frac{480}{360} = \cos\alpha$$

$$\frac{-7r + 480}{360} = \cos\alpha$$

$$\left(\frac{-7r + 480}{360}\right)^2 = \cos^2\alpha$$

$$\frac{49r^2 - 6720r + 230400}{129600} = \cos^2\alpha$$

$$\left(\frac{7}{12}r - 10\right)^2 = \left(\frac{1}{2}r\right)^2 + 10^2 - 2\left(\frac{1}{2}r\right)(10)\cos\theta$$

$$\frac{49}{144}r^2 - \frac{140}{12}r + \cancel{100} = \frac{1}{4}r^2 + \cancel{100} - 10r\cos\theta$$

$$\frac{49}{144}r^2 - \frac{1}{4}r^2 - \frac{140}{12}r = -10r\cos\theta$$

$$\frac{49}{144}r^2 - \frac{36}{144}r^2 - \frac{140}{12}r = -10r\cos\theta$$

$$\frac{13r^2}{144} - \frac{140r}{12} = -10r\cos\theta$$

$$\frac{-13r^2}{1440} + \frac{140r}{120} = r\cos\theta$$

$$\frac{-13r}{1440} + \frac{140}{120} = \cos\theta$$

$$\frac{-13r + 1680}{1440} = \cos\theta$$

$$\left(\frac{-13r + 1680}{1440}\right)^2 = \cos^2\theta$$

$$\frac{169r^2 - 43680r + 2822400}{2073600} = \cos^2\theta$$

$$\cos^2\alpha + \cos^2\theta = 1$$

Shown on next page

Theresa Alle
per.5
12-28-87

$\cos^2\alpha + \cos^2\theta = 1$

$$\frac{49r^2-6720r+230400}{129600} + \frac{169r^2-43680r+2822400}{2073600} = 1$$

$$16(49r^2-6720r+230400) + 169r^2 - 43680r + 2822400 = 2073600$$

$$784r^2 - 107520r + 3686400 + 169r^2 - 43680r + 2822400 = 2073600$$

$$953r^2 - 151200r + 4435200 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = r$$

$$\frac{151200 \pm \sqrt{(151200)^2 - 4(953)(4435200)}}{2(953)} = r$$

$$\frac{151200 \pm 77165.132}{1906} = r$$

$$\frac{151200 + 77165.132}{1906} = r \qquad \frac{151200 - 77165.132}{1906} = r$$

$$119.81382 = r \qquad 38.843058 = r$$

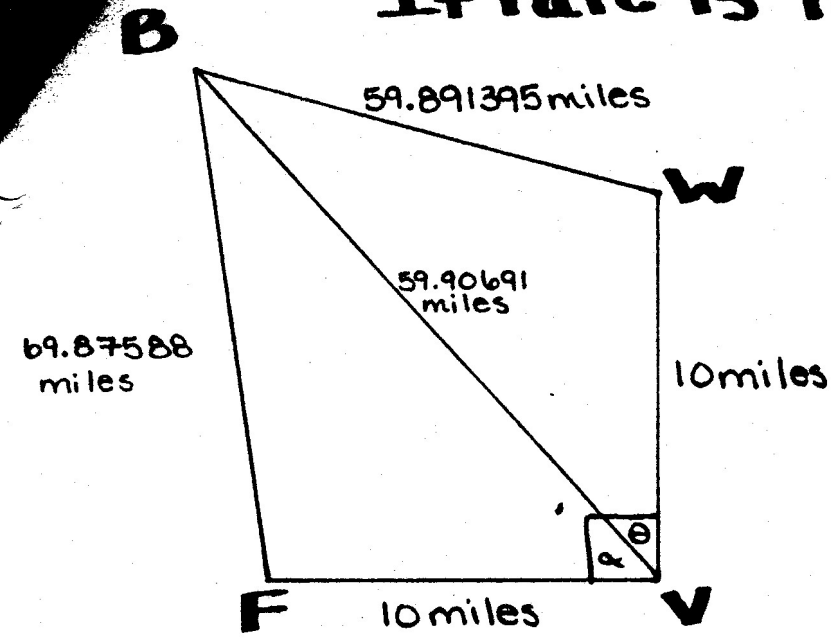
RATES COULD BE EITHER

119.81382 MPH	38.843058 MPH

THIS ONE DOESN'T WORK WHEN COSINE VALUES ARE USED TO DETERMINE ANGLE MEASUREMENT ON PAGE 4, so it should be rejected.

If rate is 119.81382 MPH

Theresa Alle
per. 5
12-28-87



$d(BW) = \frac{7}{12}r - 10$

$d(BW) = \frac{7}{12}(119.81382) - 10$

$\rightarrow d(BW) = 59.891395$ miles

$d(BV) = \frac{1}{2}r$

$d(BV) = \frac{1}{2}(119.81382)$

$\rightarrow d(BV) = 59.90691$ miles

$d(BF) = \frac{2}{3}r - 10$

$d(BF) = \frac{2}{3}(119.81382) - 10$

$\rightarrow d(BF) = 69.87588$ miles

$\cos\theta = \frac{-13r + 1680}{1440}$

$\cos\theta = \frac{-13(119.81382) + 1680}{1440}$

$\cos\theta = 0.0850141$

$\theta \doteq 85°7'$

$\cos\alpha = \frac{-7r + 480}{360}$

$\cos\alpha = \frac{-7(119.81382) + 480}{360}$

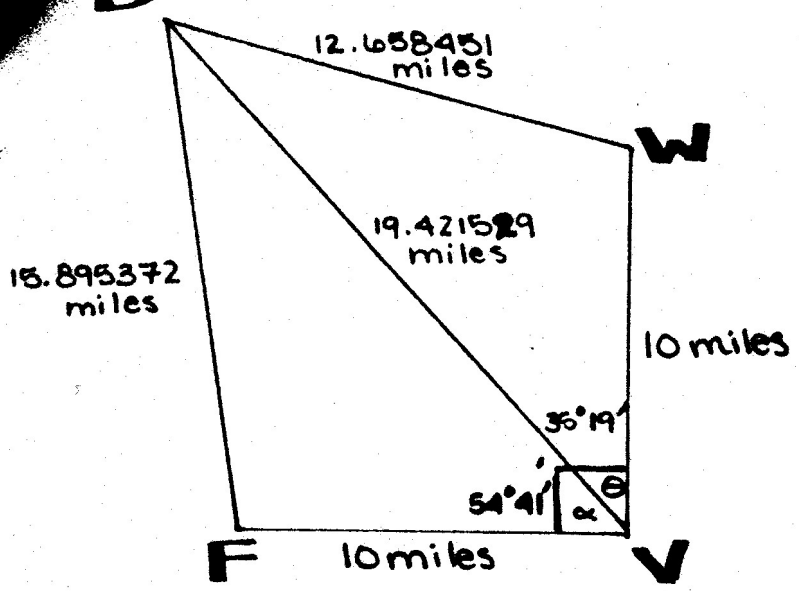
$\cos\alpha = -0.9963798$

$\alpha \doteq 175°7'$

These two angles were supposed to add to 90° according to given information as shown in pictures. They add to way over 90° (260°14') so 119.81382 MPH must be a reject for rate

If rate is 38.843058 MPH

Theresa Allen
per. 5
12-28-87



$d(BW) = \frac{7}{12}r - 10$

$d(BW) = \frac{7}{12}(38.843058) - 10$

$\rightarrow d(BW) = 12.658451$ miles

$d(BV) = \frac{1}{2}r$

$d(BV) = \frac{1}{2}(38.843058)$

$\rightarrow d(BV) = 19.421529$ miles

$d(BF) = \frac{2}{3}r - 10$

$d(BF) = \frac{2}{3}(38.843058) - 10$

$\rightarrow d(BF) = 15.895372$ miles

$\cos\theta = \frac{-13r + 1680}{1440}$

$\cos\theta = \frac{-13(38.843058) + 1680}{1440}$

$\cos\theta = 0.8160001$

$\rightarrow \theta \doteq 35°19'$

$\cos\alpha = \frac{-7r + 480}{360}$

$\cos\alpha = \frac{-7(38.843058) + 480}{360}$

$\cos\alpha = 0.57805116$

$\rightarrow \alpha \doteq 54°41'$

These two add to 90° as they are supposed to according to the given and shown in picture, so 38.843058 MPH is rate of travel

Areas found on next page

Theresa Allen
per. 5
12-28-87

Area $\Delta = \frac{1}{2} bc \sin A$

Area Δ BVF $= \frac{1}{2}(10)(19.421529) \sin 54°41'$

Area Δ BVF $\doteq 97.1076645 (0.8160)$

→ Area Δ BVF $\doteq 79.239848$ square miles

Area Δ BVW $= \frac{1}{2}(10)(19.421529) \sin 35°19'$

Area Δ BVW $\doteq 97.1076645 (0.5781)$

→ Area Δ BVW $\doteq 56.133245$ square miles

Area Quad. BWVF = Area Δ BVW + Area Δ BVF

Area Quad. BWVF $\doteq 56.133245 \text{ miles}^2 + 79.239848 \text{ miles}^2$

→ Area Quadrilateral BWVF $\doteq 135.37309$ square miles

ANSWERS

Theresa Allen
per. 5
12-28-87

Rate: 38.843058 MPH

119.81382 MPH also found but rejected as shown on page 4.

$\measuredangle \alpha$ (Lower part of $\measuredangle V$): 54°41′

$\measuredangle \theta$ (Upper part of $\measuredangle V$): 35°19′

$d(\overline{BW}) \doteq 12.658451$ miles

$d(\overline{BV}) \doteq 19.421529$ miles

$d(\overline{BF}) \doteq 15.895372$ miles

Area $\triangle BVF \doteq 79.239848$ square miles

Area $\triangle BVW \doteq 56.133245$ square miles

Area Quadrilateral BWVF $\doteq 135.37309$ miles^2

CHARLIE 5 TURKEY 3 [illegible] period 5

$BF^2 = 900x^2 + 100 - 600x\cos\theta$ $\quad$ $BW^2 = 900x^2 + 100 - 600x\cos\theta$

$$\cos^{-1}\left(\frac{BF^2 - 900x^2 - 100}{-600x}\right) + \cos^{-1}\left(\frac{BW^2 - 900x^2 - 100}{-600x}\right) = 90^\circ$$

$35 = \frac{10 + BW}{x}$ $\quad$ $40 = \frac{10 + BF}{x}$

$35x - 10 = BW$ $\quad$ $40x - 10 = BF$

$$\cos^{-1}\left(\frac{(40x-10)^2 - 900x^2 - 100}{-600x}\right) + \cos^{-1}\left(\frac{(35x-10)^2 - 900x^2 - 100}{-600x}\right) = 90^\circ$$

$$\cos^{-1}\left(\frac{1600x^2 - 800x + 100 - 900x^2 - 100}{-600x}\right) + \cos^{-1}\left(\frac{1225x^2 - 700x + 100 - 900x^2 - 100}{-600x}\right) = 9$$

$$\cos^{-1}\left(\frac{700x^2 - 800x}{-600x}\right) + \cos^{-1}\left(\frac{325x^2 - 700x}{-600x}\right) = 90^\circ$$

$$\cos^{-1}\left(\frac{8-7x}{6}\right) + \cos^{-1}\left(\frac{28-13x}{24}\right) = 90^\circ$$

$$\cos\left(\cos^{-1}\left(\frac{8-7x}{6}\right) + \cos^{-1}\left(\frac{28-13x}{24}\right)\right) = \cos 90^\circ$$

$$\left(\frac{8-7x}{6}\right)\left(\frac{28-13x}{24}\right) - \left(\sqrt{1-\left(\frac{8-7x}{6}\right)^2}\right)\left(\sqrt{1-\left(\frac{28-13x}{24}\right)^2}\right) = 0$$

$$\frac{(8-7x)(28-13x)}{144} = \left(\sqrt{1-\left(\frac{8-7x}{6}\right)^2}\right)\left(\sqrt{1-\left(\frac{28-13x}{24}\right)^2}\right)$$

$$\frac{(8-7x)^2(28-13x)^2}{20736} = \left(1-\left(\frac{8-7x}{6}\right)^2\right)\left(1-\left(\frac{28-13x}{24}\right)^2\right)$$

$$\frac{(8-7x)^2(28-13x)^2}{20736} = 1-\left(\frac{28-13x}{24}\right)^2-\left(\frac{8-7x}{6}\right)^2+\frac{(8-7x)^2(28-13x)^2}{20736}$$

$$\left(\frac{28-13x}{24}\right)^2+\left(\frac{8-7x}{6}\right)^2=1$$

$$\frac{(28-13x)(28-13x)}{576}+\frac{(8-7x)(8-7x)}{36}=1$$

$$(28-13x)(28-13x)+16(8-7x)(8-7x)=576$$

$$784-728x+169x^2+16(64-112x+49x^2)=576$$

$$784-728x+169x^2+1024-1792x+784x^2=576$$

$$953x^2-2520x+1808=576$$

$$953x^2-2520x+1232=0$$

$$\frac{2520\pm\sqrt{2520^2-4(953)(1232)}}{2(953)} \quad x=2 \text{ or } x=.647$$

$$\boxed{x=.647 \text{ miles/minute} = 38.82 \text{ mph}}$$

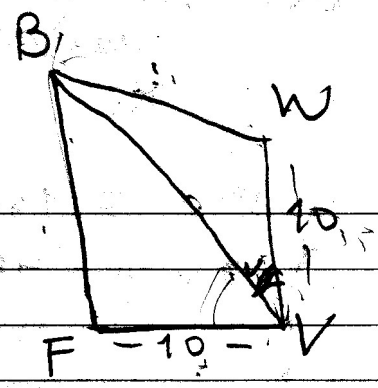
Charlie's Turkey

Helen Romelashvili

BV — 30 min $= \frac{1}{2}$ hour

BWV — 35 min $\frac{7}{12}$ hour

BFV — 40 min $\frac{2}{3}$ hour



let the speed be x then BWV is $\frac{7}{12} \cdot x = \frac{7x}{12}$ and BW is $\frac{7x}{12} - 10$.

BV is $\frac{x}{2}$

BFV is $\frac{2}{3}x$ BF is $\frac{2}{3}x - 10$

Let's write law of cosine for triangles ΔBWV and ΔBVF

$$\begin{cases} BW^2 = 100 + BV^2 - 20\,BV\cos v_1 \\ BF^2 = 100 + BV^2 - 20\,BV\cos(90 - v_1) \end{cases}$$

$$\begin{cases} \left(\frac{7x}{12} - 10\right)^2 = 100 + \frac{x^2}{4} - 10x\cos v_1 \\ \left(\frac{2}{3}x - 10\right)^2 = 100 + \frac{x^2}{4} - 10x\sin v_1 \end{cases}$$

$$\begin{cases} \frac{49x^2}{144} - \frac{7 \cdot 10}{6}x + \cancel{100} = \cancel{100} + \frac{x^2}{4} - 10x\cos v_1 \\ \frac{4x^2}{9} - \frac{40x}{3} + \cancel{100} = \cancel{100} + \frac{x^2}{4} - 10x\sin v_1 \end{cases}$$

$$\begin{cases} 49x^2 - 1680x = 36x^2 - 1440x\cos v_1 \\ 16x^2 - 480x = 9x^2 - 360x\sin v_1 \end{cases}$$

$$\begin{cases} 13x^2 = 1680x - 1440x\cos v_1 \\ 7x^2 = 480x - 360x\sin v_1 \end{cases}$$

The incredible tale of the 37-year puzzle

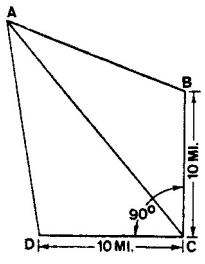
2000 readers sent solutions to a long-unsolved problem re-presented last February in "Readers Talk Back"

Darrell Huff—a frequent contributor to PS of articles on the use of pocket calculators to solve problems and play games—was a logical choice to receive the flood of puzzle responses. He's shown here organizing them, and reports that his own calculators came in handy for checking solutions. Next month, Huff starts a bimonthly column in PS called Calcu-letter.

By DARRELL HUFF

Our story has a cast of thousands, as they say in Hollywood—not to mention Howard Hughes in a bit part. The thousands have a couple of things in common. They are all readers of POPULAR SCIENCE and all determined to rescue R. F. Davis of Sun City, Ariz.

Thirty-seven years ago Dick Davis was a young writer-photographer living with a wife and baby daughter in a village near Concord, Mass. In September, 1939, he came upon a modest problem in the letters column of his new PS:



"Have enjoyed monkeying with the problems you print, for the last couple of years. Here's one that may make the boys scratch their heads a little: A man always drives at the same speed (his car probably has a governor on it). He makes it from A direct to C in 30 minutes; from A through B to C in 35 minutes; and from A through D to C in 40 minutes. How fast does he drive?

D.R.C., Sacramento, Calif."

Like a good many other readers that month—and again in February, 1977—Dick left the rest of the magazine unread for at least one evening.

The problem looked elementary—but it wasn't. Davis used every bit of math he could remember from the usual high-school courses and some meteorology encountered at Harvard. He went through his mother's old *Academic Arithmetic* and *A School Algebra* from the 1890's.

One day, many years and miles later, he found himself at the Los Angeles Public Library on a research errand for Howard Hughes, for whom he was doing public relations. On a hunch he called for the issues of POPULAR SCIENCE following the one with the puzzle. There he found one clue. This communication had appeared in November 1939:

"In regard to the problem submitted by D. R. C. of Sacramento, Calif., in the September issue. According to my calculations, the speed of the car must be 38.843 miles an hour. W.L.B., Chicago, Ill."

Without details or verification, this wasn't much help. But Davis continued to pursue the problem until last winter when, now retired in Sun City, Ariz., he capitulated after 37 years.

"All right," he wrote to Readers Talk Back. "I give up. What's the answer? Over the years I probably have shown the problem to more than 500 people . . . Now I'd like to challenge POPULAR SCIENCE. If your readers of today cannot solve this little problem (and prove the answer) how about finding the answer, editors—if there is one?"

Knowing that their own time is valuable while writers are an idle lot, the editors prudently bounced the thing my way. With considerable help from good luck and an idle thought that floated into my subconscious while waiting for a tennis court, I turned up a solution.

This solution, along with a tip on a different method I obtained from my one-time college math instructor, James Booher, emboldened the editors to publish Davis's challenge in the February issue. Their designated hitter had come up with a couple of solutions, the editors said, and "we'd like to see what you readers can do before we publish his answers."

Record-breaking replies

Over the next couple of months, "you readers could do" all this:

Throw the magazine's mail room out of joint with a record bulk of almost 2000 communications on a single subject. Produce the correct answers—yes, there are two—using at least five distinctly different approaches. (You'll find all five on the following pages.) So discommode the busy editors that they passed the whole thing along to me in large cartons, to be analyzed and reported on (see photo).

Despite the difficulty of the

Continued

we know that $X \neq 0$ so we can devide both sides of equation by X.

$$\begin{cases} 13x = 1680 - 1440\cos u_1 \\ 7x = 480 - 360\sin u_1 \end{cases}$$

$$\frac{13}{7} = \frac{168\cancel{0} - 144\cancel{0}\cos u_1}{48\cancel{0} - 36\cancel{0}\sin u_1}$$ devide by 12

$$\frac{13}{7} = \frac{14 - 12\cos u_1}{4 - 3\sin u_1}$$

$$52 - 69\sin u_1 = 98 - 84\cos u_1$$

$$84\cos u_1 = 69\sin u_1 + 46$$

$$7056(1 - \sin^2 u_1) = 4761\sin^2 u_1 + 6348\sin u_1 + 2116$$

$$7056 - 7056\sin^2 u_1 = 4761\sin^2 u_1 + 6348\sin u_1 + 2116$$

$$\rightarrow 11817\sin^2 u_1 + 6348\sin u_1 - 4940 = 0$$

$$\Delta = 68450256$$

$0 < u_1 < 90°$ $\quad \sin u_1 = \frac{-3174 + 8273.5}{11817}$ $\quad \left(\sin u_1 = \frac{-3174 - 8273.5}{11817}\right)$ is negative

$$\sin u_1 = 0.431$$

we had

$$7x = 480 - 360\sin u_1$$
$$7x = 480 - 154.8$$
$$x = 46.46$$

The answer is Charlie drove 46.46 miles per hour.

CHARLIE'S TURKEY

Oliver Fernandez
period 5

$BF^2 = 900x^2 + 100 - 600x\cos\theta$ $\qquad$ $BW^2 = 900x^2 + 100 - 600x\cos\theta$

$$\cos^{-1}\left(\frac{BF^2 - 900x^2 - 100}{-600x}\right) + \cos^{-1}\left(\frac{BW^2 - 900x^2 - 100}{-600x}\right) = 90^\circ$$

$$35 = \frac{10 + BW}{x} \qquad 40 = \frac{10 + BF}{x}$$

$$35x - 10 = BW \qquad 40x - 10 = BF$$

$$\cos^{-1}\left(\frac{(40x-10)^2 - 900x^2 - 100}{-600x}\right) + \cos^{-1}\left(\frac{(35x-10)^2 - 900x^2 - 100}{-600x}\right) = 90^\circ$$

$$\cos^{-1}\left(\frac{1600x^2 - 800x + 100 - 900x^2 - 100}{-600x}\right) + \cos^{-1}\left(\frac{1225x^2 - 700x + 100 - 900x^2 - 100}{-600x}\right) = 90^\circ$$

$$\cos^{-1}\left(\frac{700x^2 - 800x}{-600x}\right) + \cos^{-1}\left(\frac{325x^2 - 700x}{-600x}\right) = 90^\circ$$

$$\cos^{-1}\left(\frac{8 - 7x}{6}\right) + \cos^{-1}\left(\frac{28 - 13x}{24}\right) = 90^\circ$$

$$\cos\left(\cos^{-1}\left(\frac{8-7x}{6}\right) + \cos^{-1}\left(\frac{28-13x}{24}\right)\right) = \cos 90^\circ$$

$$\left(\frac{8-7x}{6}\right)\left(\frac{28-13x}{24}\right) - \left(\sqrt{1-\left(\frac{8-7x}{6}\right)^2}\right)\left(\sqrt{1-\left(\frac{28-13x}{24}\right)^2}\right) = 0$$

$$\frac{(8-7x)(28-13x)}{144} = \left(\sqrt{1-\left(\frac{8-7x}{6}\right)^2}\right)\left(\sqrt{1-\left(\frac{28-13x}{24}\right)^2}\right)$$

$$\frac{(8-7x)^2(28-13x)^2}{20736} = \left(1-\left(\frac{8-7x}{6}\right)^2\right)\left(1-\left(\frac{28-13x}{24}\right)^2\right)$$

$$\frac{(8-7x)^2(28-13x)^2}{20736} = 1-\left(\frac{28-13x}{24}\right)^2-\left(\frac{8-7x}{6}\right)^2+\frac{(8-7x)^2(28-13x)^2}{20736}$$

$$\left(\frac{28-13x}{24}\right)^2+\left(\frac{8-7x}{6}\right)^2=1$$

$$\frac{(28-13x)(28-13x)}{576}+\frac{(8-7x)(8-7x)}{36}=1$$

$$(28-13x)(28-13x)+16(8-7x)(8-7x)=576$$

$$784-728x+169x^2+16(64-112x+49x^2)=576$$

$$784-728x+169x^2+1024-1792x+784x^2=576$$

$$953x^2-2520x+1808=576$$

$$953x^2-2520x+1232=0$$

$$\frac{2520\pm\sqrt{2520^2-4(953)(1232)}}{2(953)} = x=2 \text{ or } x=.647$$

$$\boxed{x=.647 \text{ miles/minute} = 38.82 \text{ mph}}$$

B^2

$$\frac{-b - \sqrt{4ac}}{2a}$$

$$\frac{b}{2} + \sqrt{ac}$$

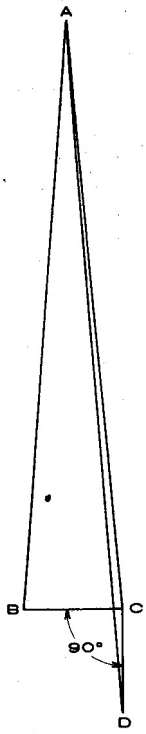
$$\frac{-6348 \pm \sqrt{6348^2 + 4(1187)(4940)}}{2(1187)}$$

problem, a solid three-quarters of those who responded came up with the right answers, usually accompanied by detailed explanations of how they got them.

These answers, carried to an extreme by one of the several solvers who had computers available, are 38.84305770085153 and 119.8138-15331676 miles per hour. Most contributors limited themselves to a reasonable eight digits or fewer, the capacity of the usual pocket calculator.

Nearly everyone who found one answer also found the second. The reason: However they started, they usually ended up with a quadratic equation. It is the nature of such an equation to have two roots, of which one often may be rejected on logical or mathematical grounds.

Some contributors vetoed nearly 120 mph on the grounds that it did not fit the figure as published (true). But many others agreed that a new figure could be drawn for that speed, meeting all requirements including the right angle. The drawing (right) shows what it looks like. Many who threw out the higher speed on non-mathematical grounds called it wasteful, illegal, or simply impossible, at least in 1939. "Obviously a fuelish person," was one comment.



E. O. Roberts, Coquitlam, B.C., says he recognized the place right away: "Airdrie to Calgary via Borden or Dicksburg. But they've put in school zones and stop signs since 1939 and now they've got a traffic light at Dicksburg. And one of them Queen's cowboys that has the yeller streaks on his legs gave me a ticket for doing 38.843 in a 30-mile zone."

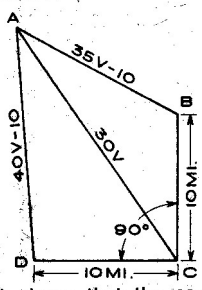
Of the few puzzle fans who reached only the 38.843 solution, most used a method called iteration. That's the hifalutin word for what was called trial and error until calculators and computers came along and made it respectable (i.e., fast and easy).

Applied to this problem, it means starting with the same equations, usually the trig ones, but bypassing a lot of involved algebra by guessing a solution and then testing it for fit. It is, as one user, R. R. Funk, Knoxville, Tenn., remarked, "a very powerful technique, not used very much, since most people have an aversion to starting with a wrong answer."

Anyone fearing that computers are about to take over the world will be comforted to note that GIGO (garbage in: garbage out)

Continued

The popular cosines



Most readers who solved the problem used some form of the law of cosines, which says that the cosine of any angle of a triangle equals the sum of the squares of the adjacent sides less the square of the other side, all divided by twice the product of the adjacent sides.

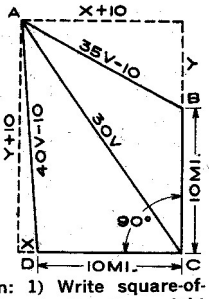
Applying this to the figure (and simplifying) shows that the cosine of the angle ABC is equal to 1/6 of 7 — 3.25v. And the cosine ACD equals 1/6 of 8 — 7v. When these two values are squared and added together they equal 1.

This follows from the rule, to be found in math books or easily derived in a few steps, that when two angles add up to 90° the sum of the squares of their cosines is 1.

Simplifying this equation and combining the terms on one side produces a quadratic equation, for which there is a standard solution that yields two roots. A convenient form of this solution that I've worked out for calculators calls for dividing the coefficients of the second and third terms by twice the coefficients of the first. Now square the first of these results and subtract twice the second from it. Find the square root. Now, separately, add this to and subtract this from that first result.

These are the two roots of the equation. Multiplied by 60 to change them from minutes to miles per hour, they become 38.843058 and 119.81382, the answers to the problem.

Construct a rectangle



Adding lines that complete a rectangle points the way to perhaps the simplest solution. After this shrewd move, only the well-known Pythagorean theorem and some standard algebra are needed for the solution. In its most direct form, the procedure is to use the given information to label the parts, as shown. Then: 1) Write square-of-the-hypotenuse equations for the three right triangles of which AC, AD, and AB are the hypotenuses. 2) Subtract the second equation from the first and find the value of x. Subtract the third from the first and solve for y. 3) Substitute these values for x and y in the first equation.

Having simplified this and divided by v^2, allowable since v can't be equal to zero (or the car wouldn't move at all), you'll have a quadratic equation. Solving this, as described above, under "The Popular Cosines," produces the answers.

Not everyone using this geometric method drew quite the same picture. A few readers preferred to draw the line parallel to DC through B instead of through A.

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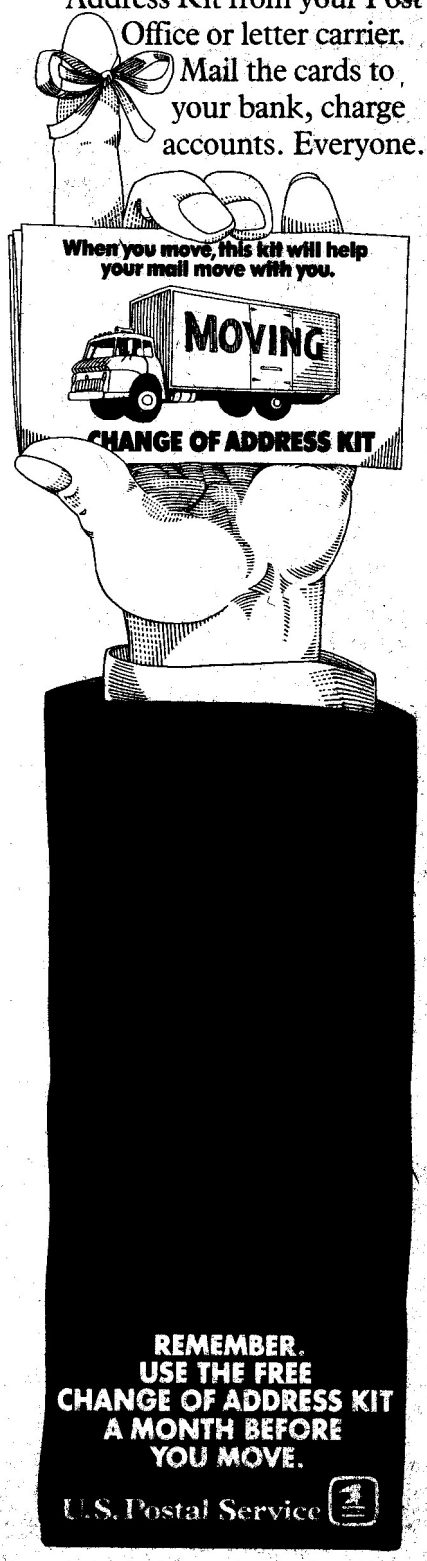

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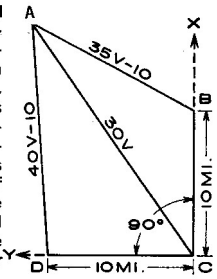
[*Continued*]

A map called analyt



Readers who had studied—and remembered—analytic geometry proceeded with a minimum of labor. What they used is formally called a system of Cartesian coordinates, which is really just a kind of map. After a reference point is chosen, all other points can be located by how far from it they are vertically and how far horizontally. When this is done for the routes in our puzzles, rules equivalent to the familiar square-of-the-hypotenuse theorem produce a set of equations.

Calling the upward direction x and the horizontal one y, the equation for the direct route AC is $x^2 + y^2 = 900v^2$. Using comparable equations for the other two routes and giving them the algebraic treatment described under "Construct a Rectangle" produces a similar quadratic equation and the same answers.

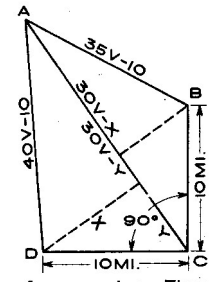
Several readers reached their initial equations in a somewhat different manner. They thought of the starting point as the intersection of the three circles having as centers the other three given points.

still holds. Thus one contributor went wrong on his initial equations and confidently submitted an impressive computer printout that unfolded to more than seven feet long, announcing that the motorist's speed was precisely 41.455297 mph.

Though reasonably close, this answer is much further off base than almost any of the ones obtained simply and quickly by readers who measured instead of calculating. These scalers made up some three percent of the contributors. Only a couple of them chose

[*Continued on page 190*]

Internal triangles

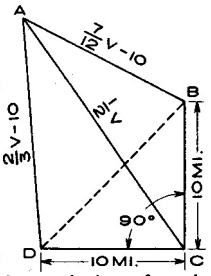


Many readers found it useful to draw lines from B and D perpendicular to AC, forming four right triangles. Using the familiar Pythagorean theorem, they easily wrote an equation for each of the upper and one of the two identical lower triangles.

They simplified each of the first two equations and solved them for x and y. They squared them, added them together, and substituted the value for $x^2 + y^2$ from the third equation. Simplifying once more brought them to a quadratic equation that could be solved as described under "The Popular Cosines."

A few readers drew a line from B to D instead and then dropped perpendiculars to it from A and C, forming a splendid collection of right triangles.

Area formulas



About one solver in a hundred ingeniously made use of area formulas: 1) Right triangle: half product of two shorter sides. 2) Any triangle: square root of product of half the sum of the sides times three quantities found by subtracting each side from half their sum. 3) Any triangle: half product of any two sides and sine of angle between them. 4) Area of any quadrilateral: subtract squares of two opposite sides from sum of squares of other two sides. Subtract square of this result from four times product of squares of the diagonals, then divide the square root of this by 4.

Most then formed a pair of equations by labeling parts in mph and setting the area of each initial triangle (by formula 2) equal to its area (by formula 3). The procedure then was much as described under "The Popular Cosines."

A variation, from Jim Knudson, Boise, Idaho, sets equal the sums of the areas of the original triangles and of the new ones formed when line AC is ignored and BD is drawn. Simplifying and substituting yields an equation solvable by iteration.

Lawrence D. Pesce of Memphis, Tenn., used the rare quadrilateral formula to set the whole area minus area BCD equal to area ABD. Squaring, substituting, and rearranging gave him the quadratic equation.

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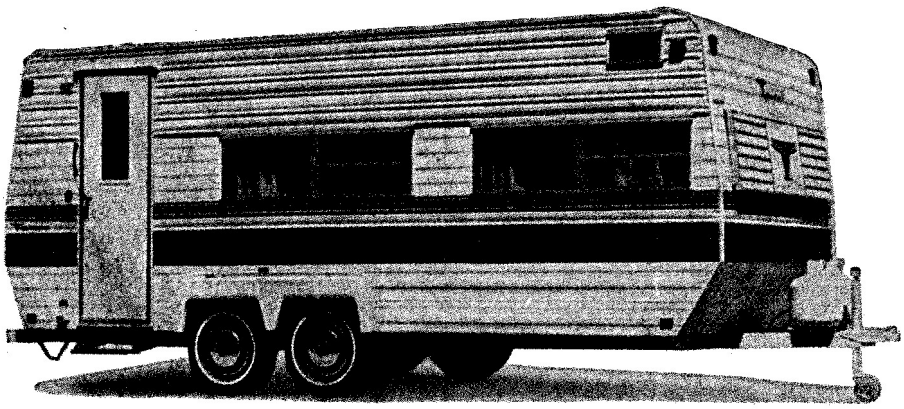

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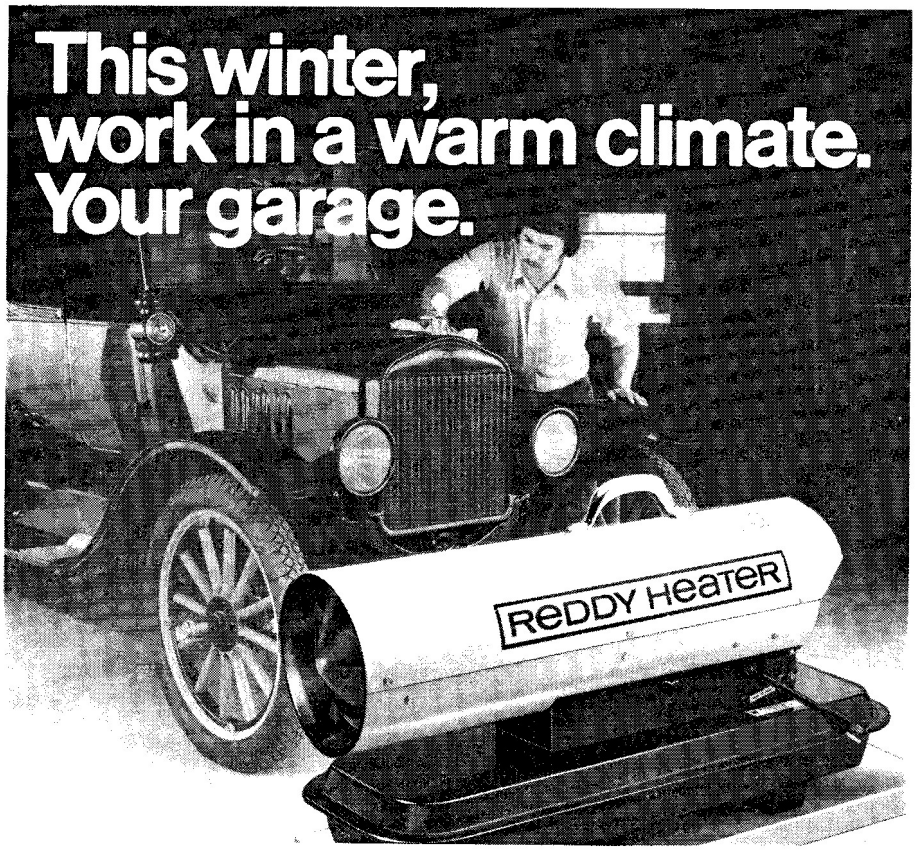

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37-year puzzle

[*Continued from page 28*]

to measure the magazine illustration, a questionable method since there was no guarantee the drawing had been made to scale. Actually it was, as close as its size permitted.

Most scalers used repeated tries with assumed distances to find the speed within one or two mph by ruler and compass. Each successive guess brought the intersections of arcs from B, C, and D closer.

H. E. Fottrell of Chattanooga found a more direct mechanical method that avoided both assumptions and repeated calculations. To two nails driven into plywood 15 inches apart, he fastened the ends of two pieces of flexible wire having lengths one-sixth and one-third greater than 15 inches. Allowing a carpenter's square to pivot against one nail, he stretched the two wires until they touched the square at equal distances from the pivot. Since this distance, which he found to be 7.75 inches, represented 10 miles, he needed only to divide one-tenth of it into twice the 15-inch length to reach a respectably accurate answer of 38.7 mph.

Trig-happy solutions

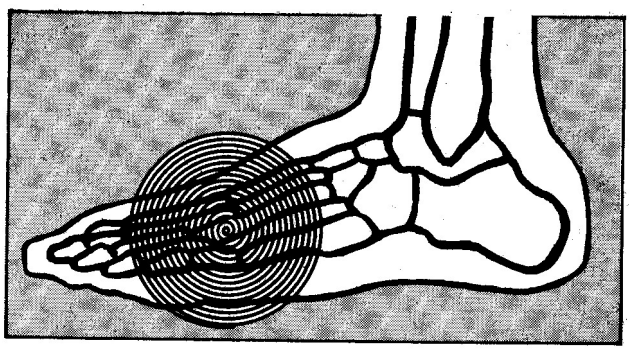
Other than scaling, the most obvious solution is by way of trigonometry. This is attested to by the fact that two-thirds of all successful solvers started with something called the law of cosines. Quite a few of these were people currently working with math, as indicated by accompanying notes, engineers' professional stationery, and—surprisingly often—use of graph paper.

About 15 percent of those who mastered the problem did so with the help of analytic geometry. Another 15 percent used plane geometry, adding construction lines to form new right triangles either inside or outside the original figure. From these initial steps, application of that most familiar of theorems (the square of the hypotenuse . . .) and some elementary algebra brought them to a quadratic equation, whose roots gave the answers.

A few rare spirits offered unique variations on these basic approaches. One of the dozen identifiably female contributors used an unusual blend of trig functions and laws. Rallying not only sines and cosines but tangents and inverse tangents as well, Stephanie Adair Vickery, La Porte, Tex., found the car's speed without needing to solve a quadratic equation.

Continued

My Feet Were Killing Me...Until I Discovered the Miracle of Hamburg!



It was the European trip I had always dreamed about. I had the time and money to go where I wanted—see what I wanted. But I soon learned that money and time don't mean much when your feet hurt too much to walk. After a few days of sightseeing my feet were killing me.

Oh, I tried to keep going. In Paris I limped through Notre Dame and along the Champs-Elysées. And I went up in the Eiffel Tower although I can't honestly say I remember the view. My feet were so tired and sore my whole body ached. While everybody else was having a great time, I was in my hotel room. I didn't even feel like sitting in a sidewalk cafe.

The whole trip was like that until I got to Hamburg, Germany. There, by accident, I happened to hear about an *exciting breakthrough for anyone who suffers from sore, aching feet and legs.*

This wonderful invention was a custom-made foot support called Flexible Featherspring. When I got a pair and slipped them into my shoes *my pain disappeared almost instantly.* The flexible shock absorbing support they gave my feet was like cradling them on a cushion of air. I could walk, stand, even run. The relief was truly a miracle.

And just one pair was all I needed. I learned that women also can wear them —even with sandals and open backed shoes. They're completely invisible.

Imagine how dumbfounded I was to discover that these miraculous devices were sold only in Europe. Right then I determined that I would share the miracle I discovered in Hamburg with my own countrymen.

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37-year puzzle

[*Continued*]

Forming the smallest minority were the four players who began with equations for the two ellipses that can be drawn through B and through D with A and C as foci. One of these elliptical solutions came all the way from Gordon D. Bredvik at Ramstein Air Base in Germany. But his was by no means the only long-distance offering. Rien Blomsma, in Rotterdam, uniquely blended construction of a rectangle with Hero's formula for the area of a triangle to find his way. Reuven Amir used a distinctive mix of sines, cosines, and tangents in a correct solution sent from Ramat Hasharon, Israel. Also using cosines were Nestor Perez (Caracas) and Rune Fredrikson (Hagerston, Sweden). Readers in Windhoek, South West Africa, and Salamanca, Spain, came croppers with 37.314 miles per hour and 27.6923 *millas per horas* respectively.

How to go wrong

Like most wrongos, these ignored the right angle stipulated between the 10-mile legs of the two alternate routes. Doing so leads to an infinity of solutions, as many noted. A fellow in Newburgh, N.Y., went to considerable trouble to program a computer to print out in two-inch letters (the T being made up of T's, and so on) the dictum that THERE IS NO ANSWER.

One thread running through much of the correspondence accompanying the solutions was that of pleasure. Pleasure of digging through old math books. Pleasure of using almost forgotten formulas and polishing up rusty old skills. And, most notably, pleasure at finding another and most stimulating exercise for their electronic calculators. ("Bless that little black box!" said a New Hampshirite.)

Not that the joy was unanimous. On stationery imprinted "A note from Pearlie," the almost-but-not-quite answer of 36⅔ mph was introduced by this plain challenge: "You men make me sick with your fancy calculators and stupid math."

There was also something a little sharp about one Canadian contribution. Signed only with initials, it was datelined Toronto, September 8, 1939. Its author expressed hope that the accompanying solution would arrive in time to be helpful. "I understand," he said, "that the United States postal service is quite slow, especially in these war times."

Chenlie's trick.

①

$(35v-10)^2 = 900v^2+100 - 2\cdot 10\cdot 30v\cos\alpha$

$1225v^2 - 700v + \cancel{100} = 900v^2 - 600v\cos\alpha$

$\dfrac{325v^2 - 700v}{-600v} = \cos\alpha$

100

Picture on pg 2.

$\cancel{13v}$

$\dfrac{28-13v}{24} = \cos\alpha.$

$(40v-10)^2 = 900v^2+100 - 2\cdot 70\cdot 30v\cos\beta$

$1600v^2 - 800v + 100 = 900v^2 + 100 - 600v\cos\beta$

$\dfrac{700v^2 - 800v}{-600v} = \cos\beta$

$\dfrac{8-7v}{6}$

$\cancel{\cos^2\alpha = \dfrac{(28-13v)^2}{24^2} = 1}$

$\alpha + \beta = 90^\circ$

$\cos(\alpha+\beta) = \cos 90$

$\cancel{\cos^2\alpha + \sin^2\beta}$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = 0$

$\cos\alpha =$

$\cos^2($

$\sin^2 x + \cos^2 x = 1$

$X + Y = 90^\circ$

$\cos^2 x + \cos^2 Y = 1$

$\sin\left(\frac{\pi}{2} - X\right) = \cos X$

$\cos^2\left(\frac{\pi}{2} - Y\right) + \cos^2 Y = 1$

$\sin\frac{\pi}{2}\cos X - \sin X \cos\frac{\pi}{2}$

0

$\left(\cos\frac{\pi}{2}\cos Y + \sin\frac{\pi}{2}\sin Y\right)^2$

$\sin^2 Y + \cos^2 Y = 1$

$\alpha = \frac{\pi}{2} - \beta$

$\frac{\pi}{4} - \alpha$

$\cos^2\alpha = \frac{(28-13v)^2}{(24)^2} = \frac{784}{6^2 \cdot 4^2}$

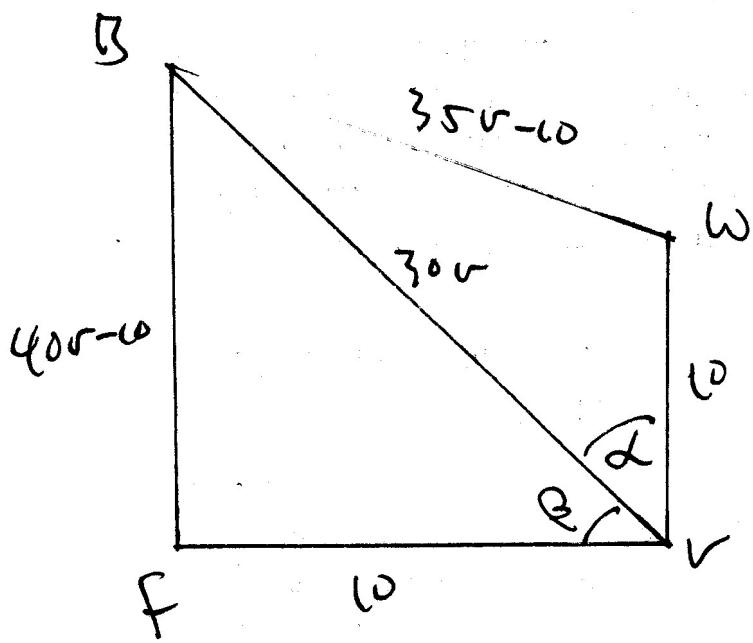
$\cos^2\beta = \frac{(8-7v)^2}{6^2} = 6^2$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\left(\frac{\pi}{2} - \beta\right) + \cos^2\alpha = 1$

$\left(\sin\frac{\pi}{2}\cos\beta - 0\right)^2 + \cos^2\alpha = 1$

$\cos^2\beta + \cos^2\alpha = 1$



$$= \frac{\text{[illegible]}}{\text{[illegible]}} + \frac{\text{[illegible]}}{\text{[illegible]}} - \frac{\text{[illegible]}}{\text{[illegible]}}$$

$$= \text{[illegible]} + \text{[illegible]} - \text{[illegible]} = \text{[illegible]}$$

$$= \frac{\text{[illegible]}}{\text{[illegible]}} + \frac{\text{[illegible]}}{\text{[illegible]}} - \frac{\text{[illegible]}}{\text{[illegible]}}$$

$$= \text{[illegible]} + \text{[illegible]} - \text{[illegible]}$$

$$= \frac{\text{[illegible]}}{\text{[illegible]}} + \frac{\text{[illegible]}}{\text{[illegible]}} - \frac{\text{[illegible]}}{\text{[illegible]}

Charlie's Turkey

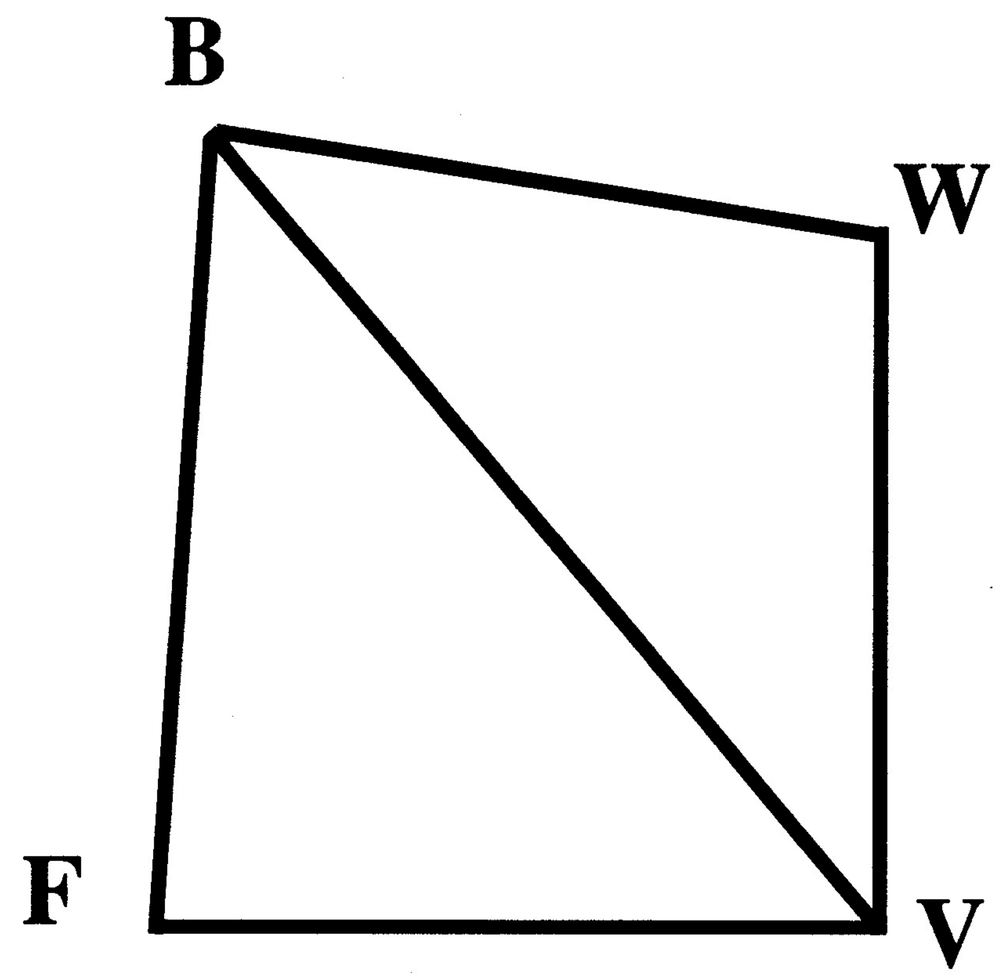
Charleton (some know him as Santa) Heston delivered turkeys to three students living in Vacaville. He delivered only one turkey on each trip and he always got his turkeys at the same place. "Just for fun", he took three different routes. Because he didn't want too much fun, he always drove at the same speed. (He had a cruise control.) See the map and six facts for assistance.

The question is: How fast did Charlie drive?

The answer to the question is due the first day we return from Christmas vacation. If you will be delayed, I will accept a solution postmarked on or before that first day we return.

Hint: This *is* enough information to answer the question. Use that new calculator that Santa brought you!

Value: 3 bonus assignments!



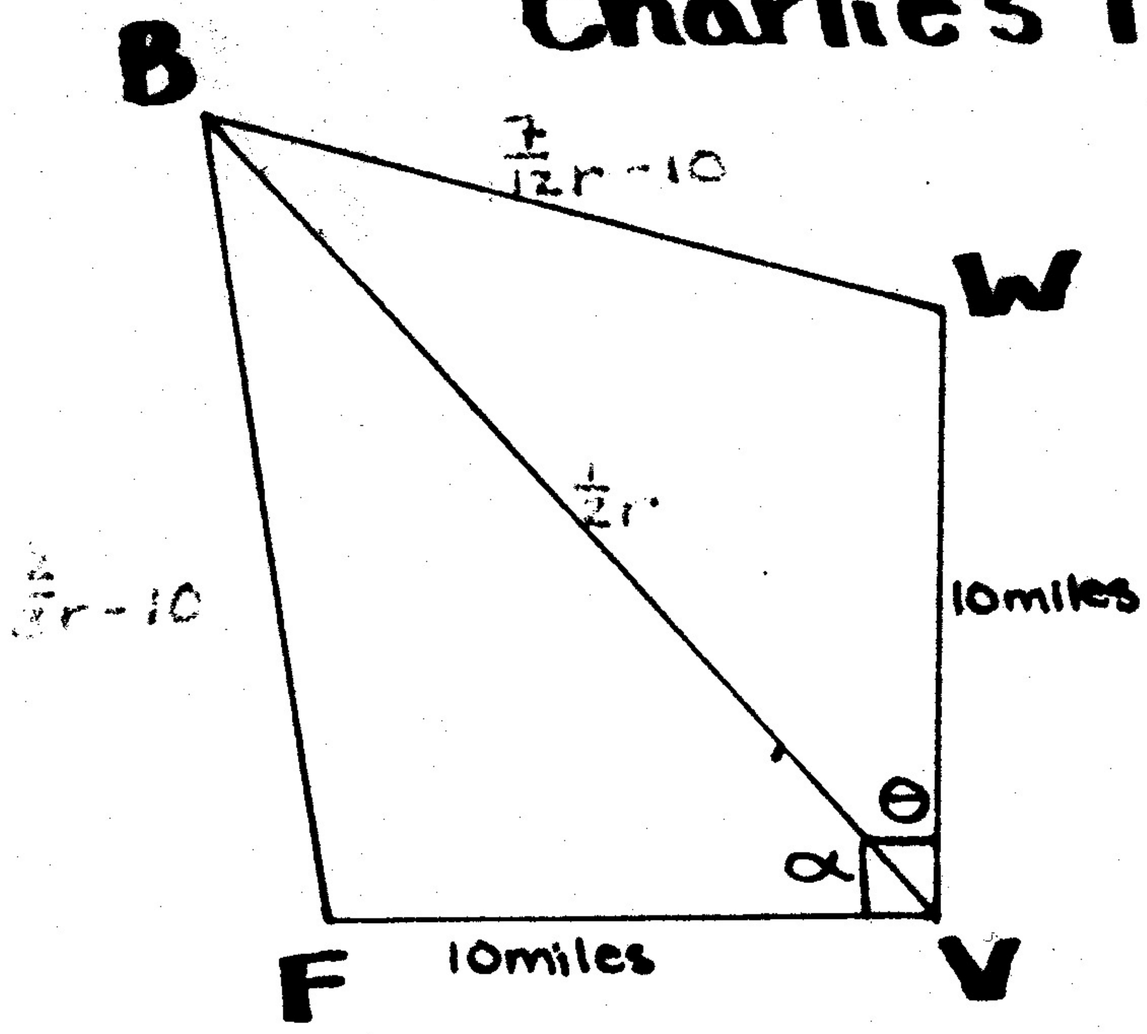
"FACTS"

1. He made it from B directly to V in 30 minutes.
2. He made it from B through W to V in 35 minutes.
3. He made it from B through F to V in 40 minutes.
4. The road V F is east - west and 10 miles long.
5. The road V W is north - south and 10 miles long.
6. All roads are perfectly straight and no speed or time is lost on any turns.

1

Charlie's Turkeys

Theresa Allen
per. 5
12-28-8



BWV takes 35 minutes or $\frac{7}{12}$ of an hour

B-V takes 30 minutes or $\frac{1}{2}$ of an hour

B-F-V takes 40 minutes or $\frac{2}{3}$ of an hour

Distance = Rate • Time

$d = rt$

$d(BWV) = \frac{7}{12}r$

$d(BV) = \frac{1}{2}r$

$d(BFV) = \frac{2}{3}r$

$d(BW) = d(BWV) - d(WV)$

$d(BF) = d(BFV) - d(FV)$

$d(BW) = \frac{7}{12}r - 10$

$d(BF) = \frac{2}{3}r - 10$

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ θ is also $(\frac{\pi}{2} - \alpha)$

$\sin^2(\frac{\pi}{2} - \alpha) + \cos^2\theta = 1$

$\sin(\frac{\pi}{2} - \alpha)^2 + \cos^2\theta = 1$

$(\underbrace{\sin\frac{\pi}{2}}_{1}\cos\alpha - \sin\alpha\underbrace{\cos\frac{\pi}{2}}_{0})^2 + \cos^2\theta = 1$

$\boxed{\cos^2\alpha + \cos^2\theta = 1}$

$\cos^2\alpha$ and $\cos^2\theta$ are found out on the next page

Theresa Allen
per. 5
12-28-87

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\left(\frac{2}{3}r - 10\right)^2 = \left(\frac{1}{2}r\right)^2 + 10^2 - 2\left(\frac{1}{2}r\right)(10)\cos\alpha$$

$$\frac{4}{9}r^2 - \frac{40}{3}r + \cancel{100} = \frac{1}{4}r^2 + \cancel{100} - 10r\cos\alpha$$

$$\frac{4}{9}r^2 - \frac{1}{4}r^2 - \frac{40}{3}r = -10r\cos\alpha$$

$$\frac{16}{36}r^2 - \frac{9}{36}r^2 - \frac{40}{3}r = -10r\cos\alpha$$

$$\frac{7}{36}r^2 - \frac{40}{3}r = -10r\cos\alpha$$

$$-\frac{7r^2}{360} + \frac{40r}{30} = r\cos\alpha$$

$$\frac{-7r}{360} + \frac{40}{30} = \cos\alpha$$

$$\frac{-7r}{360} + \frac{480}{360} = \cos\alpha$$

$$\frac{-7r + 480}{360} = \cos\alpha$$

$$\left(\frac{-7r + 480}{360}\right)^2 = \cos^2\alpha$$

$$\frac{49r^2 - 6720r + 230400}{129600} = \cos^2\alpha$$

$$\left(\frac{7}{12}r - 10\right)^2 = \left(\frac{1}{2}r\right)^2 + 10^2 - 2\left(\frac{1}{2}r\right)(10)\cos\theta$$

$$\frac{49}{144}r^2 - \frac{140}{12}r + \cancel{100} = \frac{1}{4}r^2 + \cancel{100} - 10r\cos\theta$$

$$\frac{49}{144}r^2 - \frac{1}{4}r^2 - \frac{140}{12}r = -10r\cos\theta$$

$$\frac{49}{144}r^2 - \frac{36}{144}r^2 - \frac{140}{12}r = -10r\cos\theta$$

$$\frac{13r^2}{144} - \frac{140r}{12} = -10r\cos\theta$$

$$\frac{-13r^2}{1440} + \frac{140r}{120} = r\cos\theta$$

$$\frac{-13r}{1440} + \frac{140}{120} = \cos\theta$$

$$\frac{-13r + 1680}{1440} = \cos\theta$$

$$\left(\frac{-13r + 1680}{1440}\right)^2 = \cos^2\theta$$

$$\frac{169r^2 - 43680r + 2822400}{2073600} = \cos^2\theta$$

$$\cos^2\alpha + \cos^2\theta = 1$$

Shown on next page

Theresa Allen
per. 5
12-28-87

$\cos^2\alpha + \cos^2\theta = 1$

$$\frac{49r^2 - 6720r + 230400}{129600} + \frac{169r^2 - 43680r + 2822400}{2073600} = 1$$

$$16(49r^2 - 6720r + 230400) + 169r^2 - 43680r + 2822400 = 2073600$$

$$784r^2 - 107520r + 3686400 + 169r^2 - 43680r + 2822400 = 2073600$$

$$953r^2 - 151200r + 4435200 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = r$$

$$\frac{151200 \pm \sqrt{(151200)^2 - 4(953)(4435200)}}{2(953)} = r$$

$$\frac{151200 \pm 77165.132}{1906} = r$$

$$\frac{151200 + 77165.132}{1906} = r \qquad \frac{151200 - 77165.132}{1906} = r$$

$$119.81382 = r \qquad 38.843058 = r$$

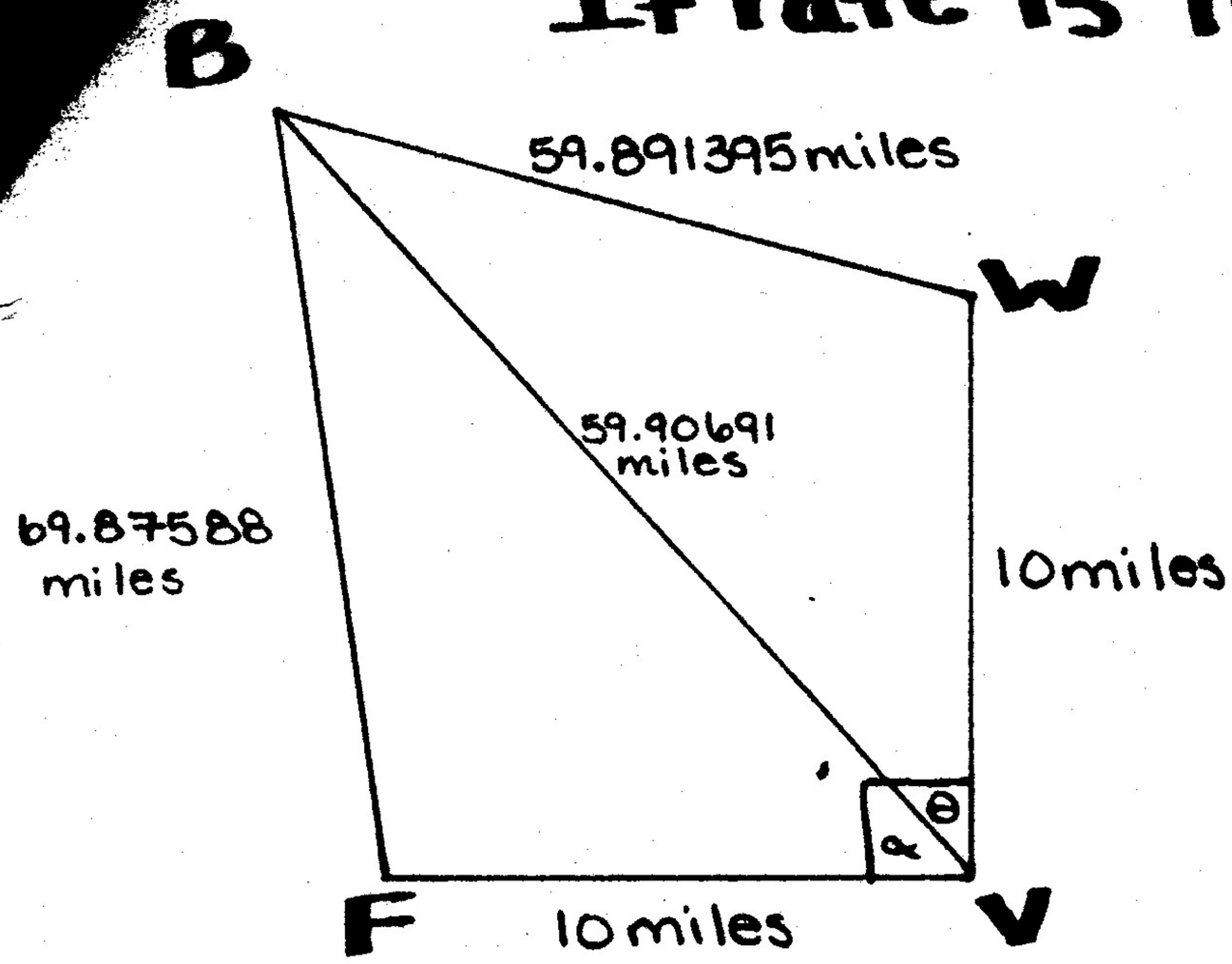
RATES COULD BE EITHER

119.81382 MPH	38.843058 MPH

THIS ONE DOESN'T WORK WHEN COSINE VALUES ARE USED TO DETERMINE ANGLE MEASUREMENT ON PAGE 4, so it should be rejected.

If rate is 119.81382 MPH

Theresa Alle
per. 5
12-28-87



$d(BW) = \frac{7}{12}r - 10$

$d(BW) = \frac{7}{12}(119.81382) - 10$

$\rightarrow d(BW) = 59.891395$ miles

$d(BV) = \frac{1}{2}r$

$d(BV) = \frac{1}{2}(119.81382)$

$\rightarrow d(BV) = 59.90691$ miles

$d(BF) = \frac{2}{3}r - 10$

$d(BF) = \frac{2}{3}(119.81382) - 10$

$\rightarrow d(BF) = 69.87588$ miles

$\cos\theta = \frac{-13r + 1680}{1440}$

$\cos\theta = \frac{-13(119.81382) + 1680}{1440}$

$\cos\theta = 0.0850141$

$\theta \doteq 85°7'$

$\cos\alpha = \frac{-7r + 480}{360}$

$\cos\alpha = \frac{-7(119.81382) + 480}{360}$

$\cos\alpha = -0.9963798$

$\alpha \doteq 175°7'$

These two angles were supposed to add to 90° according to given information as shown in pictures. They add to way over 90° (260°14') so 119.81382 MPH must be a reject for rate

If rate is 38.843058 MPH

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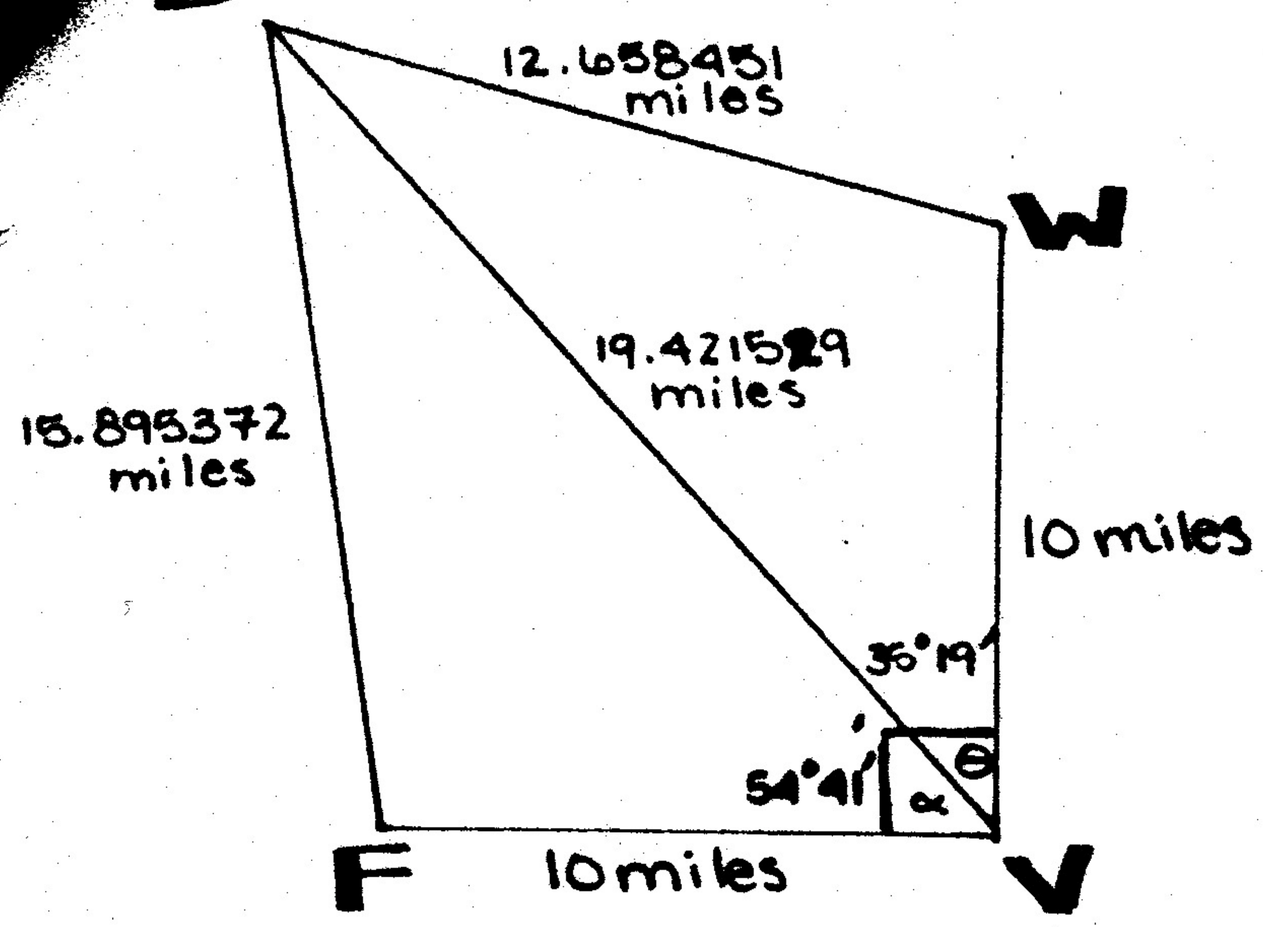
per. 5
12-28-87



$d(BW) = \frac{7}{12}r - 10$

$d(BW) = \frac{7}{12}(38.843058) - 10$

$\rightarrow d(BW) = 12.658451$ miles

$d(BV) = \frac{1}{2}r$

$d(BV) = \frac{1}{2}(38.843058)$

$\rightarrow d(BV) = 19.421529$ miles

$d(BF) = \frac{2}{3}r - 10$

$d(BF) = \frac{2}{3}(38.843058) - 10$

$\rightarrow d(BF) = 15.895372$ miles

$\cos\theta = \frac{-13r + 1680}{1440}$

$\cos\theta = \frac{-13(38.843058) + 1680}{1440}$

$\cos\theta = 0.8160001$

$\rightarrow \theta \doteq 35°19'$

$\cos\alpha = \frac{-7r + 480}{360}$

$\cos\alpha = \frac{-7(38.843058) + 480}{360}$

$\cos\alpha = 0.5786516$

$\rightarrow \alpha \doteq 54°41'$

These two add to 90° as they are supposed to according to the given and shown in picture, so 38.843058 MPH is rate of travel

Areas found on next page

Theresa Allen
per. 5
12-28-87

Area $\Delta = \frac{1}{2} bc \sin A$

Area Δ BVF $= \frac{1}{2}(10)(19.421529) \sin 54°41'$

Area Δ BVF $\doteq 97.1076645 (0.8160)$

→ Area Δ BVF $\doteq 79.239848$ square miles

Area Δ BVW $= \frac{1}{2}(10)(19.421529) \sin 35°19'$

Area Δ BVW $\doteq 97.1076645 (0.5781)$

→ Area Δ BVW $\doteq 56.133245$ square miles

Area Quad. BWVF = Area Δ BVW + Area Δ BVF

Area Quad. BWVF $\doteq 56.133245 \text{ miles}^2 + 79.239848 \text{ miles}^2$

→ Area Quadrilateral BWVF $\doteq 135.37309$ square miles

ANSWERS

Theresa Allen
per. 5
12-28-87

Rate: 38.843058 MPH

119.81382 MPH also found but rejected as shown on page 4.

$\measuredangle \alpha$ (Lower part of $\measuredangle V$): 54°41′

$\measuredangle \theta$ (Upper part of $\measuredangle V$): 35°19′

$d(\overline{BW}) \doteq 12.658451$ miles

$d(\overline{BV}) \doteq 19.421529$ miles

$d(\overline{BF}) \doteq 15.895372$ miles

Area $\triangle BVF \doteq 79.239848$ square miles

Area $\triangle BVW \doteq 56.133245$ square miles

Area Quadrilateral BWVF $\doteq 135.37309$ miles

Honors Trigonometry **Imagine That!** *Mr. Mumaugh*

I Rewrite the complex number in "coordinate form". Plot the point.

a) $3+2i$ b) $-7+6i$ c) $-\frac{8}{3}-\frac{5}{3}i$ d) $\sqrt{2}-7i$

e) $(1-i)(1+i)$ f) $(1-2i)(1+i)$ g) $2i\left(3-\frac{5}{2}i\right)$ h) $\frac{4i}{3}(-6-3i)$

II Find the absolute value (see pg 249):

a) $|5-12i|$ b) $|2i|$ c) $|1+\sqrt{2}i|$ d) $|2-3i|$

e) $|-12i|$ f) $|4-3i|$ g) $|i^7|$ h) $|3^4|$

III We indicate a complex number using letters like z, z_k (where k is usually an integer), or w etc.
z indicates a complex number of the form $a+bi$ where a is called the real part of the complex number and b (and **not** $b\,i$!) is a real number and is called the imaginary part of the complex number .
$\bar{z}$ is called the conjugate of z and is equal to $a-bi$.

a) Give an example of complex numbers z and w such that $|z+w|\neq|z|+|w|$.

b) In $z=3-4i$, find $|z|^2$, $\bar{z}z$, $\bar{z}^2z^2$

IV a) See the example in the box on the right. Give a detailed, line-by-line explanation for the example . Do something similar for parts (b) - (g).

b) $|z|=4$ c) $|z|=1$

d) $|z-2|=4$ e) $|z+3|=1$

f) $|z-2i|=4$ g) $|z-3i+2|=3$

$$|z-1|=4$$
$$|x+yi-1|=4$$
$$\sqrt{(x-1)^2+y^2}=4$$
$$(x-1)^2+y^2=4^2$$
Circle center at (1,0)
radius 4

V Express the number in ρ cis θ form:

a) $3+4i$ b) $-4+3i$ c) $5-12i$ d) $-\sqrt{7}-3i$

e) $1+2i$ f) $3-5i$ g) $-\frac{5}{2}+\frac{7}{2}i$ h) $\sqrt{5}+\sqrt{11}i$

VI Perform the indicated multiplications or division; express your answers in both rectangular form $a+b\,i$ and in polar form $\rho(\cos\theta+i\sin\theta)$.

a) $3\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)\,2\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$ b) $3\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)\,12\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)$

c) $12\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)\,\frac{7}{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ d) $\dfrac{8\left(\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}\right)}{4\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)}$

e) $\dfrac{6\left(\cos\frac{7\pi}{20}+i\sin\frac{7\pi}{20}\right)}{4\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)}$ f) $\dfrac{\sqrt{54}\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{\sqrt{6}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)}$

VII Convert to polar form and then multiply or divide. Express your answers in $\rho \operatorname{cis} \theta$ form.

a) $(1+i)(1+\sqrt{3}i)$ b) $(1-i)(3-3i)$ c) $\dfrac{(1+i)}{(1-i)}$ d) $\dfrac{(2-2i)}{(-1-i)}$

e) $3i(2\sqrt{3}+2i)$ f) $\dfrac{-4i}{(\sqrt{3}+i)}$ g) $i(1+i)(-\sqrt{3}+i)$ h) $(1-i)(2\sqrt{3}-2i)(-4-4\sqrt{3}i)$

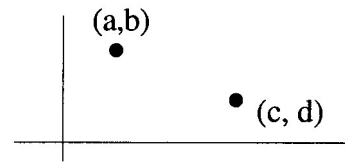
VIII The sum of two distinct complex numbers, $a+b\,i$ and $c+d\,i$, can be found geometrically by means of the so-called *parallelogram rule*: Plot the points $a+b\,i$ *and* $c+d\,i$ in the complex plane and form the parallelogram, three of whose vertices are 0, $a+b\,i$ and $c+d\,i$. Then the fourth vertex of the parallelogram is the point whose coordinate is the sum of $a+b\,i$ and $c+d\,i$.

Assume $a \neq 0$ and $c \neq 0$.

a) Find the slope of the line K from 0 to $a+b\,i$ (Note: K contains the points $(0, 0)$ and (a, b).)

b) Find the slope of the line N from 0 to $c+d\,i$.

c) Use the point-slope formula $y-y_1=m(x-x_1)$ to find the equation of the line parallel to N through (a, b). Call that line L.



d) Find the equation of the line M, through $c+d\,i$ and parallel to line K.

e) Label the lines K, L, M, N.

f) Show by using substitution that the point $(a+c, b+d)$ satisfies both the equations of line L and of line M. Therefore $(a+c, b+d)$ lies on both lines. Since the only point on both lines is the fourth vertex of the parallelogram, this vertex must be $(a+c, b+d)$, hence this vertex has coordinate $(a+c)+(b+d)\,i = (a+bi)+(c+d\,i)$.

IX Simplify. Write answers in $a+b\,i$ format.

a) $\left(\operatorname{cis}\frac{\pi}{12}\right)^6$ b) $\left(\operatorname{cis}\frac{\pi}{5}\right)^{20}$ c) $\left(3\operatorname{cis}\frac{7\pi}{30}\right)^5$ d) $\left(\sqrt[3]{4}\operatorname{cis}\frac{7\pi}{36}\right)^{12}$

e) $(2+2i)^{12}$ f) $\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)^{10}$ g) $\left(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^{20}$ h) $\left(-\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}i\right)^{14}$

X Find the n n^{th} roots of the given number in polar form:

a) fourth roots of unity b) sixth roots of unity c) $64\left(\operatorname{cis}\frac{\pi}{5}\right)$; $n=3$

d) $8\left(\operatorname{cis}\frac{\pi}{10}\right)$; $n=3$ e) $81\left(\operatorname{cis}\frac{\pi}{12}\right)$; $n=4$ f) $16\left(\operatorname{cis}\frac{\pi}{7}\right)$; $n=5$

g) -1 ; $n=5$ h) -1 ; $n=7$ i) i ; $n=5$

j) $-i$; $n=5$ k) $1+i$; $n=2$ l) $1-\sqrt{3}i$; $n=3$

XI Solve the given equation in the complex number system:

a) $x^6=-1$ b) $x^6+64=0$ c) $x^3=i$

d) $x^4=i$ e) $x^3+27i=0$ f) $x^6+729=0$

g) $x^4=-1+\sqrt{3}i$ h) $x^4=-8-8\sqrt{3}i$ i) $x^8=1$

XII Sketch the graph of the equation (Your grapher might help). Rewrite the equation in cartesian form.

a) $\rho=1-\sin\theta$ b) $\rho=3-3\cos\theta$ c) $\rho=-2\cos\theta$

d) $\rho=\cos 3\theta$ e) $\rho=\sin 3\theta$ f) $\rho=-6\sin\theta$

g) $\rho=\cos 2\theta$ h) $\rho=\sin 4\theta$ i) $\rho^2=4\cos 2\theta$

j) $\rho^2=\sin 2\theta$ k) $\rho=2+4\cos\theta$ l) $\rho=4\cos\theta+4\sin\theta$

m) $\rho=4+2\sec\theta$ n) $\rho=4\tan\theta$ o) $\rho=\sin\theta\tan\theta$

p) $\rho=\dfrac{10}{2+3\sin\theta}$ q) $\rho=\dfrac{4}{2-4\cos\theta}$ r) $\rho=\dfrac{10}{4-3\sin\theta}$

II 4

$z_1\ (3,-2)$

$z_2\ (4,1)$

$z_3 = z_1 + z_2$

$= (7,-1)$

$z_4 = z_1 z_2$

$12+2,\ 3-8$

$(14,-5)$

$z_5\ \frac{3,-2}{4,1}\ \frac{4-1}{4-1}$

$\frac{12-2}{17},\ \frac{-8-3}{17}$

$\left(\frac{14}{17}, \frac{-11}{17}\right)$

$z_6\ (3,-2)(3,-2)$

$9-4\ -12$

$(5,-12)$

$z_7\ (4,1)^3(4,1)$

$(16-1,\ 8)$

$(15,8)(4,1)$

$60-8\ \ 32+15$

$(52,\ 47)$

IV)

$z_1 = 3\operatorname{cis} 120$

$z_2 = 5\sqrt{2}\operatorname{cis} 45$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)(5,5)$

$-\frac{15}{2} - \frac{15\sqrt{3}}{2},\ \frac{15\sqrt{3}}{2} - \frac{15}{2}$

$z_4\ -\frac{15}{2}(1+\sqrt{3}),\ \frac{15}{2}(\sqrt{3}-1)$

$z_4 = 15\sqrt{2}\operatorname{cis} 165$

$(-20.490,\ 5.490)$

$z_5 = \frac{3}{5\sqrt{2}}\operatorname{cis} 75$

$(.140,\ .410)$

$z_6 = 9\operatorname{cis} 240$

$-\frac{9}{2},\ -\frac{9\sqrt{3}}{2}$

$z_7 = 125\cdot 2\sqrt{2}\operatorname{cis} 135$

$250\sqrt{2}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$(-250,\ 250)$

$\frac{1}{z} = \frac{1}{r(\cos\theta + i\sin\theta)}$

$\frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$

VI

$8\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$\left(8 \text{ cis } 30 + 360k\right)^{\frac{1}{3}}$

$2 \text{ cis } 10 + 120k$

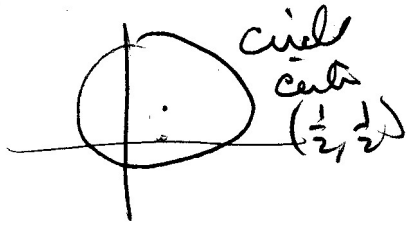
$2 \text{ cis } 10$ $\quad 1.970, \ .347$

$2 \text{ cis } 130$ $\quad -1.286, \ 1.532$

$2 \text{ cis } 250$ $\quad -.684, \ -1.879$

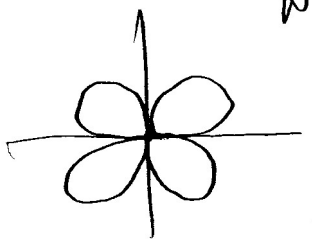
VII

a $\rho = \sin\theta + \cos\theta$

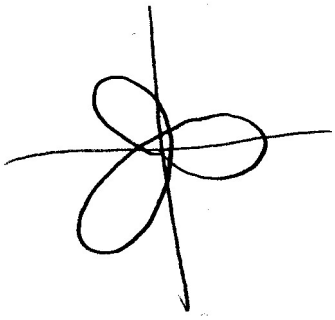


b) $\rho^2 = \sin 2\theta$

Not std graph chosen.



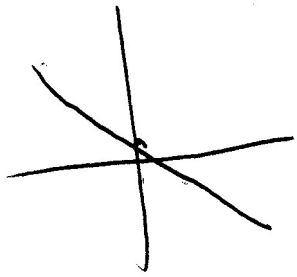
c) $\rho = \cos 3\theta$



d) $\rho = 2\sec\theta + 2\csc\theta$

$2x + 2y = 1$

$y = -x + \frac{1}{2}$



$$\ell = \frac{2}{\cos\theta + \sin\theta}$$

$$\rho = \frac{2}{\cos\theta} + \frac{2}{\sin\theta}$$

$$1 = \frac{1}{x} + \frac{1}{y}$$

$$xy = y + x$$

$$y = \frac{x}{x-1}$$

Honors Trigonometry

Copy original problem.

Convince *me* that **you** understand the concept!

Round decimals to the nearest thousandth.

Name Ch 7 Rev

Per _____ Date ~~Key~~

TRIG Tic Needs Some Repairs

Last Regular Exam ...make-up

I Given z_1 and z_2 find : $z_3 = z_1 + z_2$

A. $z_1 = 3 - 2i$, $z_2 = 4 + i$

B. $z_1 = 3 \text{ cis } 120^\circ$, $z_2 = (5, 5)$

write in (a,b) form No Decimals

write in ρ cis θ form

~~Write final~~ answers in coordinate form.

$z_4 = z_1 z_2$

$z_5 = \dfrac{z_1}{z_2}$

$z_6 = (z_1)^2$

$z_7 = (z_2)^3$

A

3 → (7, −1)

4 → (14, −5)

5 → $\left(\frac{10}{17}, -\frac{15}{17}\right)$

6 → (5, −12)

7 → (52, 47)

B

$\left(\frac{10\sqrt{2}-3}{2}, \frac{3\sqrt{3}+10\sqrt{2}}{2}\right)$

$\left(\frac{15}{2}(\sqrt{6}-\sqrt{2}), \frac{15}{2}(\sqrt{6}+\sqrt{2})\right)$

$\left(\frac{3}{40}(\sqrt{6}-\sqrt{2}), \frac{3}{40}(\sqrt{6}+\sqrt{2})\right)$

$\left(-\frac{9}{2}, -\frac{9\sqrt{3}}{2}\right)$

$(-500\sqrt{3}, 500\sqrt{2})$

~~Use no decimals in this section.~~ remove this.

(total ~~30~~ 30 pts)

II Show that if $z = \rho(\cos\theta + i\sin\theta)$, $z \neq 0$ then $\dfrac{1}{z} = \dfrac{1}{\rho}\left(\cos(-\theta) + i\sin(-\theta)\right)$ (~~15~~ 10 pts)

$\frac{1}{z} = \frac{1}{\rho(\cos\theta + i\sin\theta)} \cdot \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$

$= \frac{1}{\rho} \frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta}$

$\frac{1}{\rho}(\cos\theta - i\sin\theta)$

$\frac{1}{\rho}(\cos(-\theta) + i\sin(-\theta))$

Cos is no neg Sine is added

III Find all the cube roots of $z = 4\sqrt{3} + 4i$. Write your answers in $a + bi$ format. (~~15~~ 20 pts)

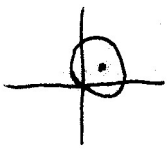
$2 \text{ cis } 10 \to 1.970 + .347i$

$2 \text{ cis } 130 \to -1.286 + 1.532i$

$2 \text{ cis } 250 \to -.684 - 1.879i$

IV Make a table containing at least six coordinates for each equation. Graph the equation on the supplied axis. Locate and label these points on your graph. Convert the equation into its cartesian form.

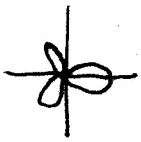
(10 ea)



a) $\rho = \sin\theta + \cos\theta$ $\quad x^2 + y^2 = x + y$

b) $\rho^2 = \sin 2\theta$ $\quad x^2 + y^2 = 2\sin x \cos x$

$(x^2+y^2)^2 = xy$



c) $\rho = \cos 3\theta$

$\rho = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= \cos^2\theta \sin\theta - \sin^3\theta - 2\sin^2\theta\cos\theta$

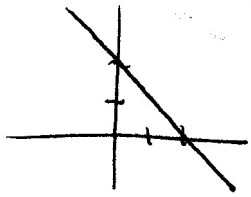
$\rho^4 =$

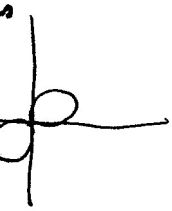
$(x^2+y^2)^2 = x^2 y - y^3 - 2xy^2$

d) $\rho = 2\sec + 2\csc\theta$

$x + y = 2$ change this to

$\rho = \dfrac{2}{\cos\theta + \sin\theta}$





I

$z_1 = 3 - 2i$

$z_2 = 4 + i$

a) $z_3 = z_1 + z_2$

$= 7 - i$

$(7, -1)$

b) $z_4 = z_1 z_2$

$= (3, -2)(4, 1)$

$= 12 + 2, 3 - 8$

$= (14, -5)$

c) $z_5 = \frac{z_1}{z_2}$

$= \frac{(3, -2)(4, -1)}{(4, 1)(4, -1)}$

$= \left(\frac{10}{17}, \frac{-11}{17}\right)$

d) $z_6 = z_1^2$

$= (3, -2)^2$

$= (3, -2)(3, -2)$

$= (5, -12)$

e) $z_7 = z_2^3$

$= (4, 1)^3$

$= (4, 1)^2 (4, 1)$

$= (15, 8)(4, 1)$

$= (52, 47)$

II

$z_1 = 3 \operatorname{cis} 120$

$z_2 = 5, 5$

$= 5\sqrt{2} \operatorname{cis} 45$

a) $z_4 = z_1 z_2$

$= 15\sqrt{2} \operatorname{cis} 165$

b) $z_5 = \frac{z_1}{z_2}$

$= \frac{3 \operatorname{cis} 120}{5\sqrt{2} \operatorname{cis} 45}$

$= \frac{3\sqrt{2}}{10} \operatorname{cis} 75$

c) $z_6 = z_1^2$

$= (3 \operatorname{cis} 120)^2$

$= 9 \operatorname{cis} 240$

d) $z_7 = z_2^3$

$= (5\sqrt{2} \operatorname{cis} 45)^3$

$= 250\sqrt{2} \operatorname{cis} 135$

III

$z = \rho(\cos\theta + i \sin\theta)$

$\frac{1}{z} = \frac{1}{\rho(\cos\theta + i\sin\theta)} \cdot \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$

$= \frac{1}{\rho}\left[\frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta}\right]$

$= \frac{1}{\rho}\left[\cos\theta - i\sin\theta\right]$

— or —

$z = \rho \operatorname{cis} \theta$

$z^{-1} = (\rho \operatorname{cis} \theta)^{-1}$

$= \rho^{-1} \operatorname{cis}(-\theta)$

$= \frac{1}{\rho}(\cos -\theta + i \sin -\theta)$

$= \frac{1}{\rho}(\cos\theta - i\sin\theta)$

cos is even, sin odd.

NOT CONVINCING
NOT explained
−8

IV

$z = 4\sqrt{3} + 4i$

$8\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= 8 \operatorname{cis}(30 + 360k)$

$z^{\frac{1}{3}} = 2 \operatorname{cis}(10 + 120k)$

$= 2 \operatorname{cis} 10 = 1.970 + .347i$

$= 2 \operatorname{cis} 130 = -1.286 + 1.532i$

$= 2 \operatorname{cis} 250 = -.684 - 1.879i$

V A) $\rho = \sin\theta + \cos\theta$

$\rho^2 = \rho\sin\theta + \rho\cos\theta$

$x^2+y^2 = y+x$

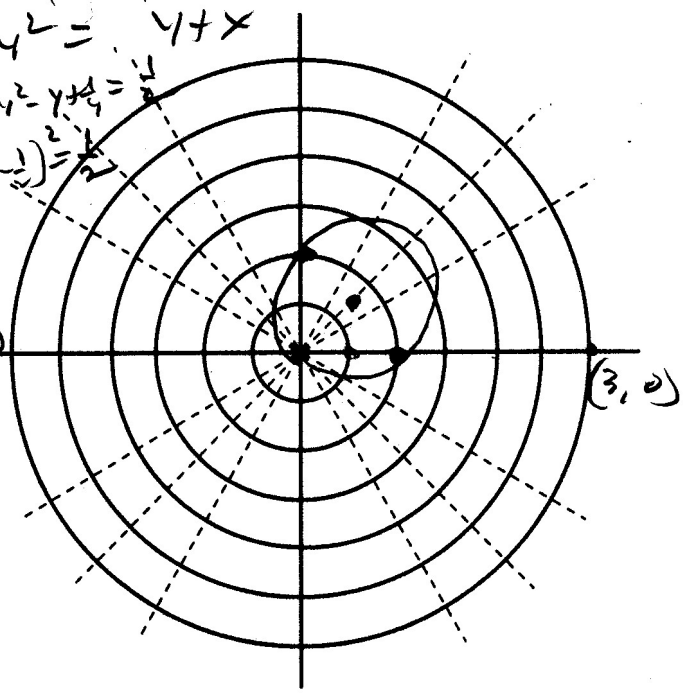
$x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} = \frac{1}{2}$

$(x-\frac{1}{2})^2 + (y-\frac{1}{2})^2 = \frac{1}{2}$

circle

radius $\frac{1}{\sqrt{2}}$

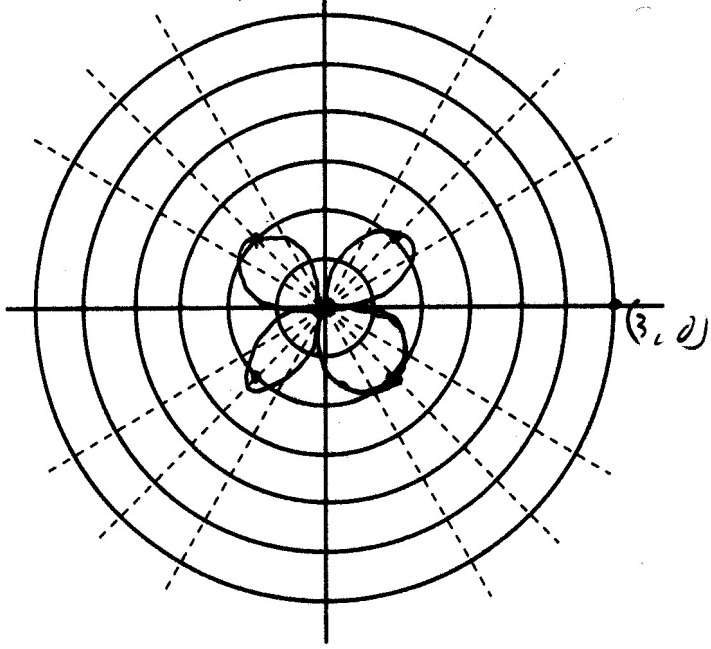
center $(\frac{1}{2}, \frac{1}{2})$



V B) $\rho^2 = \sin 2\theta$

$\rho^4 = 2\rho\sin\theta\,\rho\cos\theta$

$(x^2+y^2)^2 = 2xy$



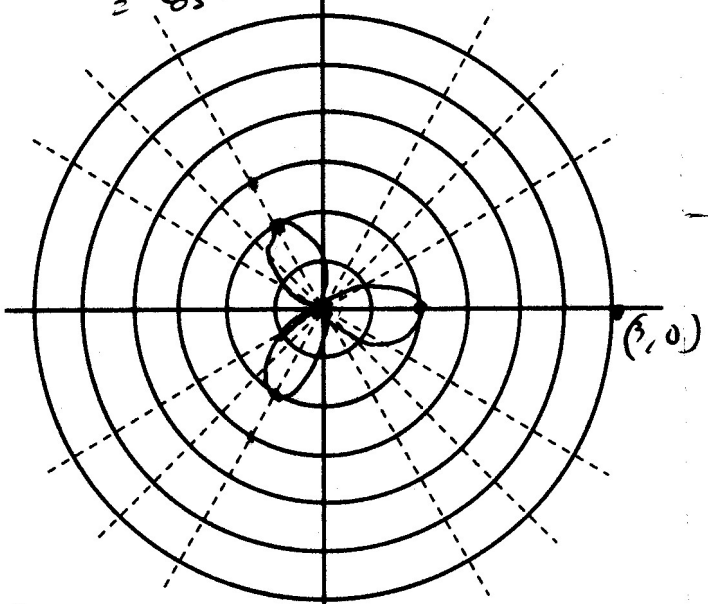
V C) $\rho = \cos 3\theta$

$\rho = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$



$\rho^4 = \rho^3\cos^3\theta - 3\rho^2\sin^2\theta\,\rho\cos\theta$

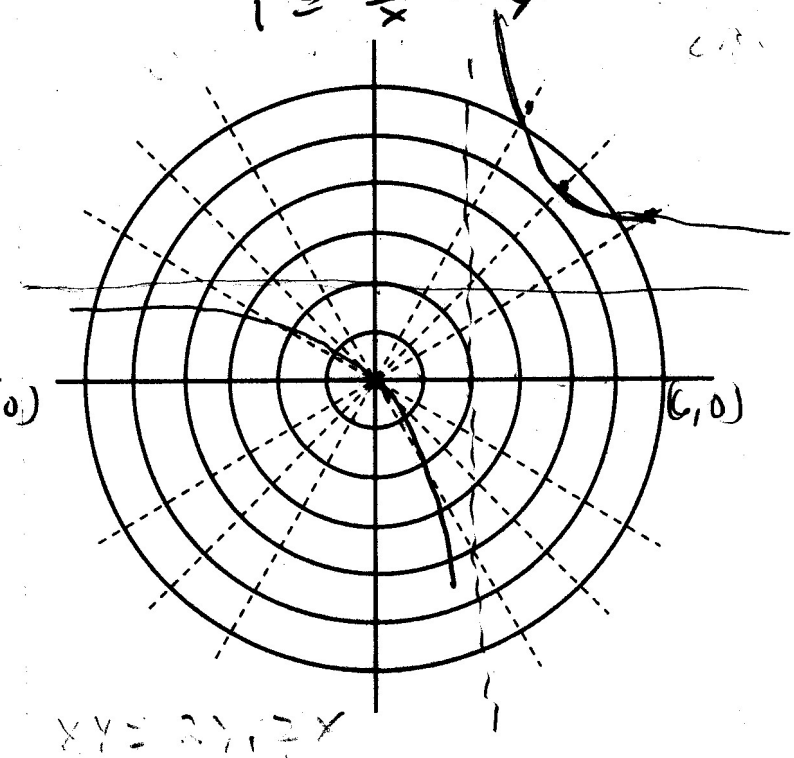
$(x^2+y^2)^2 = x^3 - 3xy^2$

V D) $\rho = 2\sec\theta + 2\csc\theta$

$\rho = \frac{2}{\cos\theta} + \frac{2}{\sin\theta}$

$1 = \frac{2}{\rho\cos\theta} + \frac{2}{\rho\sin\theta}$

$1 = \frac{2}{x} + \frac{2}{y}$



$xy = 2y + 2x$

$xy - 2y = 2x$

$(x-2)y = 2x$

$y = \frac{2x}{x-2}$

1/21/00

Robert Delgado 88
Julie Weep 87
Jeff Mohr 86
Amanda Rucker 67

No changes.

1/19/01
BJ Kent 92
Chad Gregerson 88

1/17/03
Tony Fracchia 52

Honors Trigonometry

Copy original problem.

Convince *me* that **you** understand the concept!

Round decimals to the nearest thousandth.

Name Trig ch 7

Per _____ Date Make up Exam

Last Regular Exam ...make-up

I Given : $z_1 = 3 - 2i$, $z_2 = 4 + i$

Compute the requested values.
Write answers in (a,b) format. No decimals. (20 pts tot)

a) $z_3 = z_1 + z_2$ b) $z_4 = z_1 z_2$ c) $z_5 = \frac{z_1}{z_2}$ d) $z_6 = (z_1)^2$ e) $z_7 = (z_2)^3$

Given: $z_1 = 3\text{cis}120^\circ$, $z_2 = (5,5)$

Compute the requested values.
Write answers in $\rho\text{cis}\theta$ format. No decimals. (20 pts tot)

II a) $z_4 = z_1 z_2$ b) $z_5 = \frac{z_1}{z_2}$ c) $z_6 = (z_1)^2$ d) $z_7 = (z_2)^3$

III Show that if $z = \rho(\cos\theta + i\sin\theta)$, $z \neq 0$ then $\frac{1}{z} = \frac{1}{\rho}(\cos(\theta) - i\sin(\theta))$ (10 pts)

IV Find all the cube roots of $z = 4\sqrt{3} + 4i$. Write your answers in $a + bi$ format. (20 pts)

V Make a table containing at least six coordinates for each equation. Graph the equation on the supplied axis. Locate and label these points on your graph. Convert the equation into its cartesian form. (30 tot)

a) $\rho = \sin\theta + \cos\theta$ b) $\rho^2 = \sin 2\theta$

c) $\rho = \cos 3\theta$ d) $\rho = 2\sec + 2\csc\theta$